On the weighted quadrature approach for isogeometric analysis

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ABSTRACT

The process of forming Galerkin matrices represents one of the main computational efforts in currently available isogeometric software. Indeed, the majority of isogeometric codes inherit a finite element architecture, which adopt an element-wise assembly loop with element-wise standard Gaussian quadrature. The total cost of this approach is $O(Np^{3d})$ FLOPS, where N is the number of elements, p is the spline degree, and d is the dimension of the problem. This cost clearly makes the use of high degree splines unfeasible in practical applications.

In [1], a new method to assemble these matrices was presented, which does not use the element-wise assembling loop. Instead, the method loops over the matrix rows and employs a specifically designed weighted quadrature (WQ) rule for each row.

In WQ, the integrals are approximated by incorporating the test function in the integration weights while the trial function, the geometry parametrization and the PDEs coefficients form the integrand function. For each row, the quadrature weights are chosen to satisfy some exactness conditions, whose choice is crucial for guaranteeing the optimal order of approximation.

The last ingredient of our approach is sum-factorization, which allows a significant saving of computations by exploiting the tensor structure of the spline basis functions. The computational cost of the overall strategy is $O(Np^{d+1})$ FLOPS.

The aim of this talk is to describe the main features of the method and to present some recent advances, both on the theoretical and on the computational level.

REFERENCES

[1] Calabrò, F., Sangalli, G. and Tani, M. *Fast formation of isogeometric Galerkin matrices by weighted quadrature*. Comput. Methods Appl. Mech. Engrg. (2017), **316**:606–622.