Isogeometric Analysis for Hyperbolic Conservation Laws using a Discontinuous Galerkin Method

Régis Duvigneau*, Asma Gdhami† and Maher Moakher†

* Université Côte d'Azur, Inria, CNRS, Lab. J.A. Dieudonné
Inria, Acumes Project-Team, 2004 route des lucioles, 06902 Sophia-Antipolis, France
e-mail: regis.duvigneau@inria.fr, web page: http://team.inria.fr/acumes/

† Laboratoire de Modélisation Mathématique et Numérique dans les Sciences de l'Ingénieur, ENIT
BP 37, 1002 Tunis Le Belvédère, Tunisie
Email: asmagdhami@yahoo.fr - maher.moakher@enit.rnu.tn - Web page: http://www.lamsin.rnu.tn/

ABSTRACT

Isogeometric analysis method [1] has known a growing interest in engineering fields for the last years, thanks to its ability to strengthen the coupling between Computer Aided Design (CAD) and Finite-Element Analysis (FEA). However, this methodological framework is not so well suited to hyperbolic conservation laws, encountered in problems governed by wave propagation phenomena, such as acoustics, electromagnetics or compressible aerodynamics. In this context, additional stabilization terms may be required [2], whose calibration is tedious.

Therefore, we investigate in this work the adaption of the Discontinuous Galerkin (DG) method [3] to handle geometries defined by Non-Uniform Rational B-Spline (NURBS) boundaries. More specifically, we explain how to generate a DG-compliant computational domain from a set of NURBS boundaries, i.e. a set of rational elements allowing discontinuities in the solution at the interfaces, while relying on an exact description of the boundaries geometry. The proposed approach is based on a combination of classical CAD procedures, such as degree elevation and knot insertion. Then, a DG solving procedure is derived, which uses the same basis for the geometry and solution according to the isogeometric paradigm, and includes a more straightforward stabilization through upwind fluxes.

The approach is applied to different hyperbolic conservation law problems, ranging from linear advection to compressible Euler equations. Optimal convergence rates of the method are demonstrated, while the critical role of boundary curvature is reported.

REFERENCES

[1] J.A. Cottrell, T.J.R. Hughes and Y. Bazilevs, *Isogeometric analysis : towards integration of CAD and FEA,* John Wiley & sons, 2009

[2] T.J.T. Hughes, G. Scovazzi and T.E. Tezduyar, "Stabilized methods for compressible flows", *J. Sci. Comput.*, vol. 43 (2010).

[3] J.S. Hestaven and T. Warburton, *Nodal Discontinuous Galerkin Methods: Algorithms, Analysis and Applications,* Springer, 2008.