Linear Dependence of Minimal Support B-splines over Box-partitions

Francesco Patrizi[*], Tor Dokken[*] and Vibeke Skytt[*]

[*]SINTEF Digital, Department of Mathematics and Cybernetics
P.O. Box 124 Blindern, 0314 oslo, Norway
e-mail: Francesco.Patrizi@sintef.no, web page: http://www.sintef.no/math

ABSTRACT

Locally refined splines such as LR B-splines, T-splines and Hierarchical B-splines have attracted much attention in the last decade as the spline spaces can be refined locally when extra degrees of freedom are needed. Thus the large increase in degrees of freedom resulting from refinement of tensor product B-spline spaces can be avoided.

An attractive and important property of univariate B-splines is minimal support, a property that is essential in the refinement process for LR B-splines. All B-splines resulting from an LR B-spline refinement process will be minimal support. This is opposed to T-spline refinement and Hierarchical B-splines refinement where B-splines will not be minimal with respect to the underlying box-partition.

When the work started on LR B-splines one idea investigated was to identify all minimal support B-splines in the Box-partition rather than explicitly performing the LR B-spline refinement process. When investigating this idea it was soon detected that it is simple to create examples where the collection of LR B-splines resulting from the refinement process were linearly independent while the collection of the minimal support B-splines in the associated Box-partition was linearly dependent.

To understand when linear dependence of LR B-splines can occur we study configurations of 2-varaite Box-partitions where the collection of minimal support B-splines are linearly dependent. We then assess if such configuration of Box-partitions can be obtained through the LR mesh construction process. Then we check if the collection of LR B-splines produced are linearly dependent by using the hand-in-hand principle. In this way, we check if the dimension of the LR spline space matches the corresponding number of the minimal support B-splines spline on such a box-partition for each step of the refinement process.

We have identified a number of configuration of 2-variate Box-portions where the collection of minimal subset B-splines is linearly dependent. Only a subset of these can be reproduced by LR B-spline refinement.

REFERENCES

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