

# Multipatches formulation for Reissner-Mindlin shells with reduced quadrature rules in IGA

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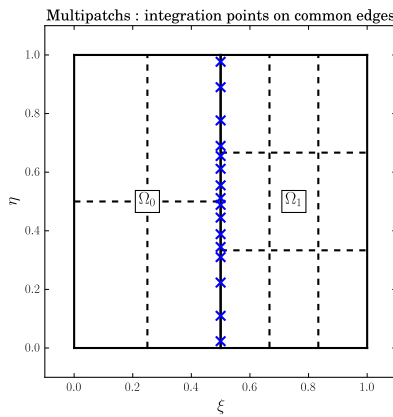
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To describe the complexity of parts used in an industrial problem, multipatches formulations are essential in the context of isogeometric analysis. The present work focus on Reissner-Mindlin based shells with rotational degrees-of-freedom . The high regularity provided by BSplines functions cannot remove numerical locking. To prevent this phenomena, a reduced quadrature rule is employed on each patches (see [1]) but also at the common interfaces between them.

Three methods in development will be proposed at the conference. The first two methods are based on penalty and Lagrange multipliers approach to weakly enforce  $L^2$  continuity (see [2]). The last one is based on TSplines to create a global conforming patch. These methods should apply for all kinds of mesh and with different parametrisations between two patches at a common edge. To illustrate this, an example for the penalty approach is proposed. Let  $\mathcal{V}_{\alpha,h}$  be the discrete space of patch  $\alpha$ ,  $\Gamma_c$  a common interface and  $\langle \cdot, \cdot \rangle$  the traditional inner product. The problem can be formulated as : find  $\{\underline{u}, \underline{\theta}\} \in \mathcal{V}_{1,h} \times \mathcal{V}_{2,h}$  with  $\underline{\chi} = \underline{u}^{(1)} - \underline{u}^{(2)}$  and  $\underline{\zeta} = \underline{\theta}^{(1)} - \underline{\theta}^{(2)}$  on  $\Gamma_c$  such that

$$a_p(\underline{u}, \underline{v}) = a(\underline{u}, \underline{v}) + \underbrace{\beta_1 \langle \underline{\chi}, \underline{\Psi} \rangle_{L^2, \Gamma_c}}_{\text{displacement}} + \underbrace{\beta_2 \langle \underline{\zeta}, \underline{\mu} \rangle_{L^2, \Gamma_c}}_{\text{rotation}} = L(\underline{v}), \forall \{\underline{v}, \underline{\vartheta}\} \in \mathcal{V}_{1,h} \times \mathcal{V}_{2,h},$$

where  $\beta$  are the penalty factors (norm  $H^{1/2}$  for the jump across the displacement),  $\underline{\Psi} = \underline{v}^{(1)} - \underline{v}^{(2)}$  and  $\underline{\mu} = \underline{\vartheta}^{(1)} - \underline{\vartheta}^{(2)}$  on  $\Gamma_c$ . In order to integrate the penalized terms, different quadrature rules on  $\Gamma_c$  will be proposed to prevent shear locking. First, an exact mesh intersection is done to compute a common knot vector on  $\Gamma_c$ . Then, a line integration which uses both orders is set.



Full Gauss-Legendre - sum of the two orders

The left figure shows an example of classical full Gauss-Legendre quadrature rule. Patch  $\Omega_0$  has  $h_0 = 2$  elements per side, order  $p_0 = 1$  and  $C^0$  regularity whereas patch  $\Omega_1$  has  $h_1 = 3$  elements per side, order  $p_1 = 2$  and  $C^1$  regularity. Number of Gauss points is defined as

$$n_{GP} = E_{sup} \left[ \frac{2(p_{\Omega_0} + p_{\Omega_1}) + 1}{2} \right]$$

A reduced integration rule could lead to integrate with two points on the boundaries of the common edge and only one point on the interior of  $\Gamma_c$ . Membrane and bending locking will be observed, depending on the order of the chosen basis.

For the dual approach, a special attention to the Lagrange multipliers space will be done. An enrichment in the energy terms will also be proposed to penalize the jumps on the efforts along the interfaces.

## Références

- [1] C. Adam, T.J.R. Hughes, S. Bouabdallah, M. Zarroug and H. Maitournam. *Selective and reduced numerical integration for NURBS-based isogeometric analysis*. Comput. Methods Appl. Mech. Engrg., vol. 284, pages 732-761, 2015.
- [2] A. Apostolatos, R. Schmidt, R. Wüchner, K.-U. Bletzinger. *A Nitsche-type formulation and comparison of the most common domain decomposition methods in isogeometric analysis*. Int. J. Numer. Meth. Engng, vol. 97, pages 473-504, 2014.