On the relation of scaled boundary IGA and classical Galerkin IGA

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ABSTRACT

This contribution is concerned with a new numerical method to solve the elasticity problem for solids in boundary representation. Starting with the basic idea of the scaled boundary finite element method [1], we derive a formulation where the geometrical description of the boundary is sufficient for defining the equations of elasticity of the complete solid. This approach fits perfectly to the boundary representation modeling technique ('b-rep') commonly employed in computer aided design. For the analysis, the weak form of the equilibrium equations is first enforced for the circumferential direction. Applying the isogeometric paradigm, the NURBS functions that describe the boundary of the geometry form also the basis for the approximation of the displacement at the boundary. The displacement field in the radial scaling direction, on the other hand, is approximated by one-dimensional NURBS, and here we have the choice of using again a weak form and Galerkin projection or, alternatively, collocation. Overall, this procedure yields a linear system of equilibrium equations whose solution gives rise to the displacement response.

In the talk, the relation of this approach to the classical concept of Isogeometric Analysis (IGA) is analyzed. In particular, focusing on a linear problem, we compare classical IGA with scaled boundary IGA (SB-IGA) where the weak form and Galerkin projection are used both in scaling and in circumferential direction. By analyzing a model problem and by exploiting the product structure inside the integrals, we then derive a general relation between the two methods. In particular situations, this is convenient for studying error and convergence of SB-IGA. Moreover, as there is an exact solution of the boundary value problem in scaling direction available in SB-IGA, this can be beneficial for investigating the singularity in the scaling center and its impact on the numerical solution in classical IGA.
We remark that this work is also related to the recent paper by Mantzaflaris et al. [3] on low rank tensor approximation.

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