Efficient quadrature rules for subdivision surfaces in isogeometric analysis

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ABSTRACT

We introduce a new approach to numerical quadrature on geometries defined by subdivision surfaces based on quad meshes in the context of isogeometric analysis. Starting with a sparse control mesh, the subdivision process generates a sequence of finer and finer quad meshes that in the limit define a smooth subdivision surface which can be of any manifold topology. Traditional approaches to quadrature on such surfaces rely on per-quad integration, which is inefficient and typically also inaccurate near vertices where other than four quads meet. Instead, we explore the space of possible groupings of quads and identify the optimal macro-quads in terms of the number of quadrature points needed. We show that macro-quads consisting of quads from one or several consecutive levels of subdivision considerably reduce the cost of numerical integration. Our rules possess a tensor product structure and the underlying univariate rules are Gaussian, i.e., they require the minimum possible number of integration points in both univariate directions.

The optimal quad groupings differ depending on the particular application. For instance, computing areas, volumes, or solving the Laplace problem lead to different spline spaces with specific structure in terms of degree and continuity. We show that in most cases the optimal groupings are quad-strips consisting of $(1 \times n)$ quads, while in some cases a special macro-quad spanning more than one subdivision level offers the most economical integration.