Adaptive IGAFEM with optimal convergence rates: Hierarchical B-splines

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ABSTRACT

The CAD standard for spline representation in 2D or 3D relies on tensor-product B-splines. To allow for adaptive refinement, several extensions have emerged, e.g., analysis-suitable T-splines, hierarchical splines, or LR-splines. All these concepts have been studied via numerical experiments. However, so far there exists only little literature concerning the thorough mathematical analysis of adaptive isogeometric finite element methods (IGAFEM). Indeed, we are only aware of the work [1] which investigates an error estimator reduction of an IGAFEM with certain hierarchical splines, as well as [2] which investigates linear convergence of an IGAFEM with truncated hierarchical B-splines. However, the mathematical proof that either adaptive strategy leads to optimal convergence rates, remained open. For standard FEM with globally continuous piecewise polynomials, adaptivity is well understood; see, e.g., [3] for symmetric elliptic PDEs. In the frame of isogeometric boundary element methods (IGABEM), we also mention our recent work [4] which shows linear convergence with optimal rates for an adaptive IGABEM in 2D, where it is however sufficient to use univariate non-uniform rational B-splines (NURBS).

In this talk, we consider an IGAFEM [5] for elliptic (possibly non-symmetric) second-order PDEs in arbitrary space dimension. We employ hierarchical B-splines of arbitrary degree and different order of smoothness. We propose a refinement strategy to generate a sequence of locally refined meshes and corresponding discrete solutions, where adaptivity is driven by some weighted residual *a posteriori* error estimator. The adaptive algorithm guarantees linear convergence of the error estimator (resp. the sum of energy error plus data oscillations) with optimal algebraic rates.

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