Manifold-based finite element basis functions with sharp features and cracks

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ABSTRACT

We present an extension of the isogeometric manifold-based basis functions introduced in [1] to geometries and solution fields with sharp features, such as edges and corners, or cracks. Manifold-based surface construction techniques are well known in geometric modelling and a number of variants exist. Common to all is the concept of constructing a smooth surface by blending together overlapping patches (or, charts), as in differential geometry description of manifolds. In our implementation manifold techniques are combined with conformal parameterisations and the partition-of-unity method for deriving basis functions on unstructured quadrilateral meshes.

Each patch on the surface consists of several elements and has a corresponding planar patch with a smooth one-to-one mapping onto the surface. On the collection of conformally parameterised planar patches the partition of unity method is used for approximation. The smooth partition of unity, or blending, functions are assembled from tensor-product b-spline segments defined on a unit square. Polynomials with prescribed degree and continuity are used as local approximants on each patch. Sharp features and cracks are represented with suitably chosen C^0-continuous and discontinuous local polynomials respectively. Finally, in order to obtain a mesh-based approximation scheme the coefficients of the local approximants are expressed in terms of vertex coefficients.

Our numerical simulations indicate the optimal convergence of the resulting approximation scheme for Poisson problems and near optimal convergence for thin-plate and thin-shell problems discretised with structured and unstructured quadrilateral meshes.

REFERENCES

[1] Majeed, M. and Cirak, F. Isogeometric analysis using manifold-based smooth basis functions. *Computer Methods in Applied Mechanics and Engineering* (2017) **316**:547–567.