Subdivision surfaces with isogeometric analysis adapted refinement weights

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ABSTRACT

Subdivision surfaces provide an elegant isogeometric analysis framework for integrated geometric design, simulation and optimisation of thin-walled shell structures, see e.g. [1]. Subdivision can be viewed both as the generalisation of splines to arbitrary connectivity meshes and as a process for generating surfaces through successive refinement of given control meshes. While the first viewpoint is expedient for finite element analysis the latter has proven itself as exceptionally effective in computer graphics, animation and geometric design.

The subdivision refinement rules are usually adapted from knot insertion rules for splines. For instance, the quadrilateral Catmull-Clark scheme considered in this work is equivalent to cubic b-splines away from extraordinary, or irregular, vertices with different than four adjacent elements. Around extraordinary vertices the surface consists of a nested sequence of smooth spline patches which join C^1 continuously at the point itself. As known from geometric design literature, the subdivision weights can be optimised so that the surface quality is improved, see e.g. [2,3]. We extend the related techniques to minimisation of errors in approximating quadratic shapes as measured in thin-shell energy norm. To this end, the eigenanalysis of the subdivision matrix underlying the subdivision refinement is employed to facilitate the optimisation process. As our finite element computations confirm the optimised subdivision weights yield a reduction in discretization errors in the energy and L_2 norms. Although the convergence constants are improved the convergence rates are, as to be expected, the same as for the classical Catmull-Clark weights.

REFERENCES

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