Isogeometric skeleton-penalty methods for flow problems: fitted and unfitted settings

T. Hoang[1,2], C.V. Verhoosel[1], G.J. van Zwieten[1], F. Auricchio[3], E.H. van Brummelen[1], and A. Reali[3]

[1]Energy Technology and Fluid Dynamics,
Department of Mechanical Engineering,
Eindhoven University of Technology, 5600MB Eindhoven, The Netherlands
e-mail: {t.hoang, c.v.verhoosel, g.j.v.zwieten, e.h.v.brummelen}@tue.nl

[2] Scuola Universitaria Superiore IUSS, 27100 Pavia, Italy
e-mail: tuong.hoang@iusspavia.it

[3] Department of Civil Engineering and Architecture,
University of Pavia, 27100 Pavia, Italy
email: {auricchio, alessandro.reali}@unipv.it

ABSTRACT

An Isogeometric Skeleton-Penalty method with equal-order approximations is proposed for incompressible flow problems. The study consists of two parts: the first part is for a fitted isogeometric setting [1], and the second part is for a trimmed isogeometric setting [2, 3].

In the first part, we introduce the key idea of the method: stabilizing the jumps of high-order derivatives of pressure and velocity fields over the skeleton of the mesh. For B-splines basis functions of degree k with C^{k-1} regularity, only the k th-order derivative is controlled. The second part is an extension of the first part into the unfitted setting. It is known that when applied to trimmed geometries, common approaches such as (isogeometric) Babuska-Brezzi stable velocity-pressure pairs and Galerkin-least square type stabilization techniques exhibit local oscillations in the vicinity of the cut boundaries [4]. We show that by using the proposed technique developed from the fitted setting, oscillation-free solutions are recovered. Optimal convergence rates are obtained.

Numerical experiments are performed for Stokes and Navier-Stokes equations, with both manufactured solutions and benchmark problems. An application to fluid flow in porous media with CT scan-based data in 3D is also illustrated.

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