On spurious pressure modes in divergence-conforming discretisation for Stokes-problem

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ABSTRACT

To solve the incompressible flow problems using isogeometric analysis, the div-compatible spline spaces were originally introduced by Buffa, de Falco and Sagnalli [1], and later further developed by Evans [2]. Johannessen, Kumar and Kvamsdal [3] extended these developments to adaptively meshes using LR B-splines as introduced by Johannessen, Kvamsdal and Dokken [4].

While having a number of attractive properties, including *point-wise* divergent free solutions, the technique produced a number of spurious pressure modes. The exact number and character of these modes depend on the choice of boundary conditions, but for no-slip conditions on all edges on a 2D rectangular (parametric) domain, it consisted of 4 corner pressure modes, in addition to the average pressure. Multiple techniques was proposed to address these modes, arguably the most popular is to circumvent the problem by weakly enforcing tangential velocity. In this talk we suggest a method to strongly enforce no-slip (or prescribed velocity) to the set of these spaces. The method generalizes to mixed Dirichlet/Neumann boundaries, non-rectangular geometries and volumes. It is strongly consistent, stable and provides optimal convergence rates in both velocity and pressure. The proposed method consist of sampling the momentum equations in their strong form and can be seen as a generalization of collocation methods, when only applied to the boundary.

We will demonstrate the suitability of the proposed boundary constraints for 2D tensor surface, 3D tensor volume and 2D LR splines cases with analytical solution.

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