Efficient isogeometric preconditioners for Stokes system

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ABSTRACT

We consider the problem of finding efficient preconditioners for Stokes system in the framework of isogeometric analysis. This is a difficult task, when high degree B-splines are employed for the discretization of the problem. Following [1], efficient preconditioning strategies involve the creation of good approximations of the stiffness block and of the Schur complement block. To address this issue, we exploit the tensor product structure that is present in isogeometric discretization and we develop a preconditioning method that is based on the solution of a Sylvester-like equation at each step. This strategy is inspired by the main ideas and results present in [2]. In our analysis we use different types of discretization spaces. First we investigate the use of isogeometric Taylor-Hood elements, that have first been studied in the context of isogeometric analysis in [3]. Then we also employ the isogeometric Raviart-Thomas elements. This last choice has the advantage of being a div-conforming discretization. The properties of this space have been deeply investigated for example in [4, 5]. For each discretization space we develop a preconditioning strategy that is robust with respect to spline degree, i.e. it is p scalable. We investigate also the consequences of the presence of highly complicated geometries, that may affect the good behaviour of the strategies. To deal with this problem, we propose different approaches based on approximations of the geometry effects on the solving equations.

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