Local Truncation Error and Dispersion for Isogeometric Method Solving Flexural Wave Equation

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ABSTRACT

The Isogeometric Method solves the flexural wave differential equation in terms of a differential (in time) difference (in space) equation for semidiscretization procedure as for Finite Element and Difference methods [1,2]. The semidiscretized isogeometric differential difference equation of the Bernoulli flexural wave equation (numerical version of the wave equation) is given by:

$$\sum_{i=1}^{n} k_i v_i + \sum_{i=1}^{n} m_i \ddot{v}_i = 0 \tag{1}$$

where v_i is the deflection at node i, k_i and m_i are stiffness and mass parameters respectively and n depends on the mesh and the polynomial degree of the NURBS basis functions. Time derivative is denoted by a superscript dot. Taking into account the wave equation Eq. (2) can be written as:

$$\sum_{i=1}^{n} k_i v_i + \sum_{i=1}^{n} \frac{m_i EI}{m} v_i^{IV} = 0 \tag{2}$$

where m is the mass per unit of length and EI is the inertia product. Taylor expansion of the deflection is given by (similar for fourth derivative):

$$v_i = v_0 + v_0^{I}\left(i\Delta x\right) + v_0^{II}\frac{\left(i\Delta x\right)^2}{2} + v_0^{III}\frac{\left(i\Delta x\right)^3}{6} + v_0^{IV}\frac{\left(i\Delta x\right)^4}{24} + .. \tag{3}$$

where Δx is the mesh length. Substituting Eq. (3) into Eq. (2) the first non-vanish term define the order of the local truncation error. The dispersion is obtained substituting the numerical wave version

$$v_i = Ae^{j(\omega t - Ki)} \tag{4}$$

where ω is the wave angular frequency, t is the time variable, j is the complex unit and K is given by:

$$K = \left(2\pi\Delta x/\lambda\right)\left(c/c_n\right) \tag{5}$$

where c is the wave velocity, c_n is the numerical wave velocity and λ is the wave length resulting the cubic dispersion equation:

$$\cos^3(K) + A\cos^2(K) + B\cos(K) + C = 0 \tag{6}$$

where A, B and C are frequency dependent parameters.

Numerical results indicate that the local truncation error is of order fourth for cubic Nurbs function and the dispersion analysis showed that for any wave frequency the resultant numerical wave is evanescent (K parameter is complex).

REFERENCES

[1] G. D. Smith, *Numerical solution of partial differential equations*, 2nd Edition, Clarendon Press, 1978.

[2] J.A.Cottrell, T.J.R. Hughes, Y. Bazilevs, *Isogeometric Analysis: Toward Integration of CAD and FEA*, Wiley, 2009.