Abstract:
Mathematical Analysis of Generalized Finite Element Method

Over the past decade, the use of the Generalized Finite Element Method (GFEM) is steadily becoming popular, in the context of the numerical solution of Partial Differential Equations. In contrast to the classical Finite Element Method (FEM), the GFEM is flexible in two ways: (a) it allows the use of non-polynomial shape functions, and consequently, could be used to approximate solutions (usually non-smooth) that cannot be well approximated by piecewise polynomials; (b) in GFEM, the shape functions can be constructed either without a mesh or a simple mesh that does not have to conform to the boundary of the underlying domain, and thus, the method avoids some of the problems of meshing a complicated domain. Methods similar to GFEM are used in practice under different names, *e.g.*, the Partition of Unity Method, the eXtended Finite Element Method, Method of Clouds, etc.; they differ primarily in the shape functions employed in their implementation.

The flexibility and promise of GFEM has been effectively shown in certain important problems, primarily through computations. But a serious theoretical understanding of GFEM is necessary for its success, especially in terms of efficiency and robustness, when applied to complex engineering problems. Some mathematical analysis of GFEM is available, but the research in this area is still emerging. All the questions that were relevant in the theoretical analysis of FEM have their counterparts in the context of GFEM; we mention for example, a posteriori error estimation and adaptivity, locking, numerical integration, solution of the associated linear system etc. It is also extremely important to identify and characterize the class of problems, where the GFEM have clear advantage over classical methods.

The goal of this mini-symposium is two fold: (a) to disseminate the current mathematical results on GFEM and related methods, and (b) presentation of computational studies to illuminate the theoretical aspects of GFEM and to suggest new mathematical research directions. The mini-symposium may include, but is not restricted to, the following topics:

- Selection of approximating functions, Approximation theory, and error estimates
- Numerical integration
- Locking
- Aposteriori error estimates and adaptivity
- Solution of the linear system
- Computation of various functionals of practical interest