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TWO STEP TAYLOR-GALERKIN SOLUTION OF LAGRANGIAN EXPLICIT DYNAMIC SOLID MECHANICS

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ABSTRACT

Traditional explicit solid dynamics formulations are based on the central difference time integration of the second order dynamic equilibrium equation for the displacements [1-2]. Using linear elements this leads to second order convergence for displacements but first order convergence for strains and stresses and constant stress hexahedron with hourglass control are typically used. Perhaps more importantly, many applications involving complex realistic geometries can often only be meshed using triangles or tetrahedra. In addition, mesh adaptation is often required following large strains but can only be achieved at a reasonable cost with simple tetrahedral elements. Unfortunately, these elements lock in the presence of nearly incompressible deformations, which are common in elastoplastic flows. Efforts to develop tetrahedral elements that are effective in nearly incompressible situations and not overly stiff in bending have only been partially successful, as the resulting formulations suffer from artificial mechanisms similar to hourglassing [3-4].

Recently, a formulation based on first order conservation laws has been proposed for explicit solid dynamics using a Discontinuous spatial discretization approach [5]. This is based on formulating the problem in terms of the conservation of momentum and energy:

$$\begin{aligned}\frac{\partial \boldsymbol{p}}{\partial t} - \nabla_0 \cdot \boldsymbol{P} &= \rho_0 \boldsymbol{b} \\ \frac{\partial E}{\partial t} + \nabla_0 \cdot \left(\boldsymbol{Q} - \tfrac{1}{\rho_0} \boldsymbol{P}^T \boldsymbol{p} \right) &= 0\end{aligned} \tag{1}$$

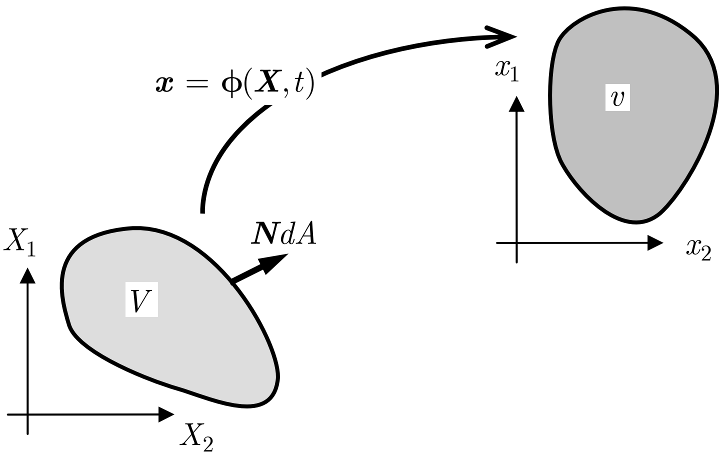
where $\boldsymbol{p} = \rho_0 \boldsymbol{v}$ is the linear momentum, E the total energy per unit initial volume, $\boldsymbol{Q}$ is the heat flux vector, $\boldsymbol{P}$ the first Piola-Kirchhoff stress tensor and $\boldsymbol{b}$ the external body forces (see figure 1); together with a further equation for the rate of change of the deformation gradient given as:

$$\frac{d\boldsymbol{F}}{dt} - \nabla_0 \cdot (\tfrac{1}{\rho_0} \boldsymbol{p} \otimes \boldsymbol{I}) = \boldsymbol{0} \tag{2}$$

This represents a mixed formulation in which both the velocities and deformation gradient are primary problem variables. In this way the interpolation accuracy for

velocities and strains/stresses will be equal. Similar conservation law formulations have been used by other authors in the Eulerian context [6-7].

The paper will present a 2-step Taylor-Galerkin discretization of the above equations [8-9] which permits the use of linear triangles and tetrahedral elements without volumetric locking or difficulties in bending applications.



$$\boldsymbol{F} = \frac{\partial \phi}{\partial \boldsymbol{X}} = \nabla_0 \boldsymbol{x};$$
$$\boldsymbol{v} = \frac{\partial \phi}{\partial t} = \dot{\boldsymbol{x}}$$
$$\boldsymbol{t} = \boldsymbol{PN};$$
$$\boldsymbol{P} = \frac{\partial \psi(\boldsymbol{F}, \theta, \alpha)}{\partial \boldsymbol{F}}$$

Figure 1: Problem definition and notation

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