

## On the stability of finite element schemes for finite strain incompressible elasticity.

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### ABSTRACT

It is well known in the literature that many finite element formulations fail to properly reproduce highly incompressible finite strain solutions, showing unphysical instability hourglass modes [1].

This has been a puzzling point for many researchers who have been devoted many efforts in trying to produce stable and robust numerical schemes also in a finite strain regime. However, up today there is no effective and exhaustive proof that any of the proposed approach is really working, due also to the fact that there is not yet a clear theoretical understanding on the numerical pathologies.

As a consequence, within the framework of incompressible finite elasticity, we focus on some simple two-dimensional examples, for which we are able to study the stability of the continuum problem. We then theoretically and numerically study the stability of the solutions to the discretized problem, obtained by means of some mixed and mixed-enhanced finite elements, which are well-performing in linear elasticity (preliminary results can be found in [2,3]).

We conclude drawing some general considerations. In particular, we find out that, in some situations, not only enhanced schemes fail in reproducing the solution stability range, but that similar failures can be also observed when using mixed finite elements generally considered reliable.

### REFERENCES

- [1] S. Reese and P. Wriggers. “A stabilization technique to avoid hourglassing in finite elasticity”. *International Journal for Numerical Methods in Engineering*, Vol. **48**, 79–109, 2000.
- [2] F. Auricchio, L. Beirão da Veiga, C. Lovadina and A. Reali. “A stability study of some mixed finite elements for large deformation elasticity problems”. *Computer Methods in Applied Mechanics and Engineering*, Vol. **194**, 1075–1092, 2005.
- [2] F. Auricchio, L. Beirão da Veiga, C. Lovadina and A. Reali. “The importance of the exact satisfaction of the incompressibility constraint in nonlinear elasticity: mixed FEMs versus NURBS-based approximations”. Submitted to *Computer Methods in Applied Mechanics and Engineering*, 2008.