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LIMIT ANALYSIS WITH ADAPTIVE MESH REFINEMENT

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ABSTRACT

Adaptive mesh procedures are widely used in conventional finite element analysis and largely eliminate the tedium of preparing quality grids by hand. This paper describes an adaptive meshing procedure for rigorous lower and upper bound limit analysis with reference to geomechanics. Unlike conventional finite element meshes, the limit analyses procedures incorporate statically and kinematically admissible discontinuities between adjacent elements. These discontinuities permit large stress and velocity jumps over an infinitesimal distance and reduce the number of elements needed to predict the collapse load accurately. In general, the role of the discontinuities is crucial as their arrangement and distribution has a dramatic influence on the accuracy of the analyses (Chen, 1975). To ensure that the stress discontinuities are positioned optimally in a lower bound grid requires an error estimator and mesh adaptation strategy which account for the presence of singularities in the computed stress field. Similarly, in an upper bound analysis, it is necessary to position the velocity discontinuities so that they capture emerging slip surfaces precisely in the collapse mechanism.

A modified form of the anisotropic mesh adaptation algorithm presented by Borges *et al.* 2001 (for a mixed limit analysis formulation) is shown to perform well for discontinuous upper bound methods, with a mix of isotropic and anisotropic refinement being better for lower bound methods. The adapted meshes typically have large numbers of discontinuities in the direction of the maximum rate of change in the stress and velocity fields. As a result, important features in a lower bound analysis, such as a fan of discontinuities centred on the point of singularity in the boundary conditions (e.g. at the edge of a strip footing), can be modelled efficiently. For upper bound analysis the anisotropic mesh adaptation results in elements being stretched along the slip surfaces.

Usually, mesh refinement proceeds with a gradual adjustment of the element size so as to distribute the local error uniformly over the mesh. Another alternative is to choose the element size distribution which minimizes the global error in some designated control variable **u** over the mesh. This approach is known as the optimal-mesh-adaptive technique and is described in detail by Almeida *et al.* 2000. In the current study we adopt an improved version of this scheme, as implemented by Lyamin *et al.* 2005, with plastic multipliers and velocities being the control variables.

The advancing front type of mesh generator has proved to be convenient for the adaptive remeshing procedures as it accounts for oriented element elongation. Indeed,

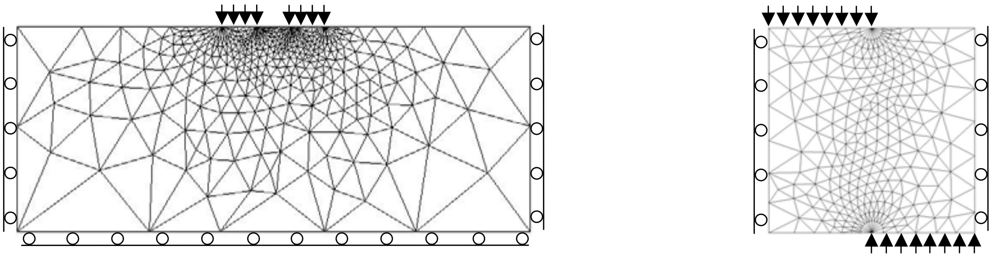
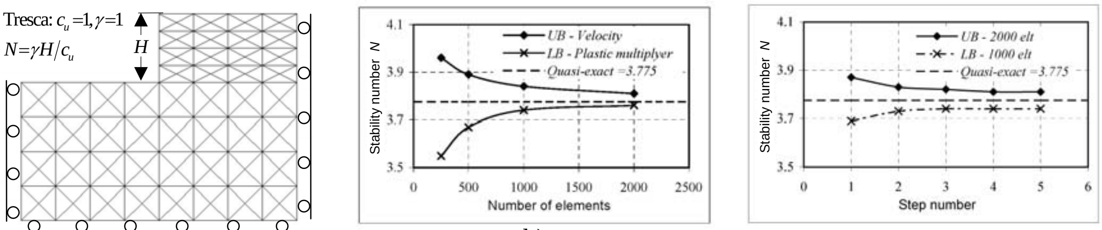


Figure 1. Examples of lower bound meshes with fan of discontinuities at singular points.

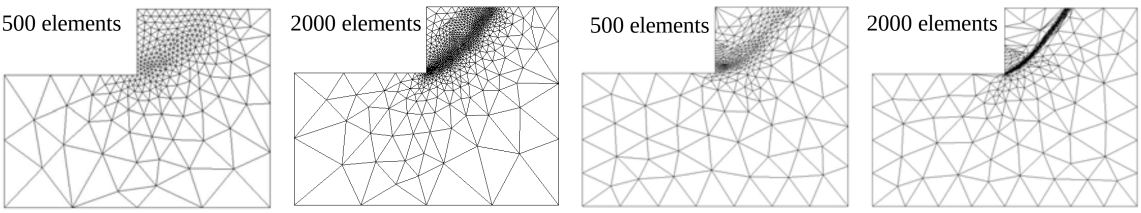
this technique is ideally suited to generating fans of elements for lower bound computations involving stress singularities. The only extra information needed is the location of the origin for each fan to be generated, the density of elements in the fan, and the threshold for switching to a normal advancing front routine. Examples of meshes with multiple fan inclusions are shown in Figure 1.



a) b) c)

Figure 2. Vertical cut: a) initial mesh, b) convergence with the number of elements, c) convergence with the number of adaptation steps.

The efficiency of the proposed adaptive remeshing scheme is demonstrated for the classical problem of the collapse of an unsupported vertical cut in purely cohesive (Tresca) soil. The initial mesh and results are presented in Figure 2. The final lower and upper bound meshes are shown in Figure 3.



a) b)

Figure 3. Adapted meshes: a) lower bound - isotropic, b) upper bound - anisotropic.

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