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VARIATIONAL METHODS FOR CONSISTENT SINGULAR AND SCALED MASS MATRICES

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Mass matrices in explicit and implicit finite element analysis mostly come along in their *consistent* or *lumped* version. Consistency in this context refers to a derivation from a given spatial discretization along with a strong functional relationship in time of displacement, velocity and momentum. In some cases, however, more freedom and flexibility in designing mass matrices is desirable. One example is the technique of mass scaling with the aim of increasing the critical time step in explicit dynamics without significant loss of accuracy [2]. Moreover, alternative mass matrices based on mass redistribution techniques may help to reduce oscillating contact forces in impact problems [1].

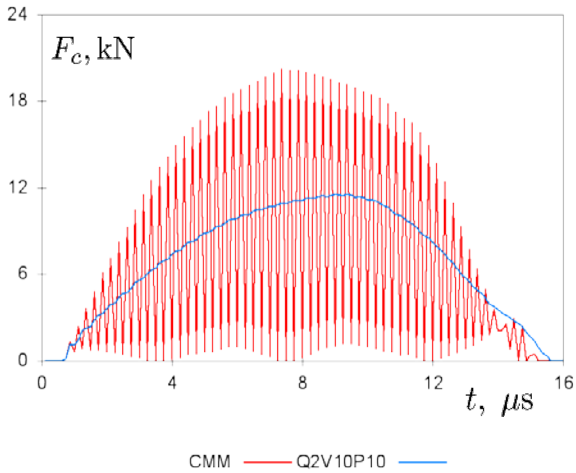
In general, these techniques lack a sound variational basis and they may be inconsistent, thus compromising convergence and accuracy. Therefore, in [3] a new variational method for formulation of consistent mass matrices has been proposed. It is based on a new penalized Hamilton's principle, where relations between variables for displacement, velocity and momentum are imposed via a penalty method. Independent spatial discretization of the variables along with a local static condensation for velocity and momentum yields a parametric family of consistent mass matrices which allows for constructions which are more flexible than classical approaches based on a one-field Hamilton's principle.

In this framework new mass matrices with desired properties can be constructed without violating conistency. It is demonstrated how such constructions may be designed to obtain

1. consistent *singular* mass matrices and
2. consistent *selectively scaled* mass matrices.

An attractive feature of formulations of the first group in implicit impact analysis is the possibility to construct singular mass matrices with zero components at those nodes where contact is collocated. This improves numerical stability of the semi-discrete problem: the differential index of the underlying differential-algebraic system is reduced from 3 to 1, and spurious oscillations in the contact pressure, which are commonly reported for formulations with Lagrange multipliers, are significantly reduced [4]. Results of numerical experiments for truss and Timoshenko beam elements are discussed. In addition, the properties of the novel discretization scheme for an unconstrained dynamic problem are assessed by a dispersion analysis.

Mass matrices of the second group are applicable to explicit analysis. It is demonstrated how usage of these non-diagonal scaled mass matrices decreases the maximum frequency of the discretized system and thus allows for larger steps in explicit time integration. At the same time the lowest eigen-frequencies in the range of interest and global structural response are not significantly changed. Results of numerical experiments for two-dimensional and three-dimensional problems are discussed. It is demonstrated how the variational framework allows designing finite element schemes with particular features. It is, for instance, possible to derive scaled mass matrices which retain the sequence of the eigen-modes compared to the standard consistent mass matrix.



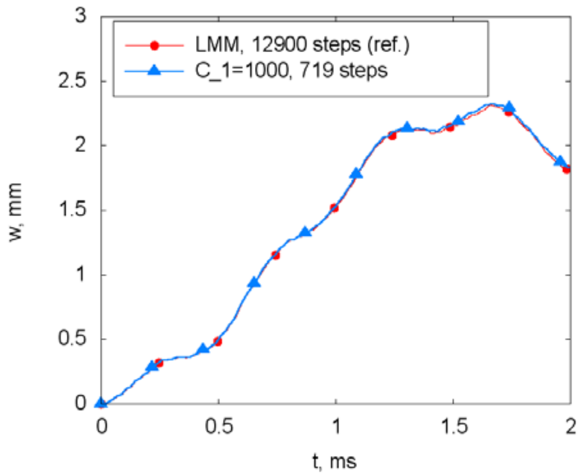


Fig. 1: left: force-time diagram, standard consistent vs. singular mass matrix, right: displacement-time diagram, lumped mass vs. varationally scaled mass

Figure 1, left, shows numerical results from a 2D plane stress analysis of the impact of a circular disc, discretized with quadratic finite elements, onto a rigid obstacle. While a standard discretization with a consistent mass matrix (CMM) exhibits wild temporal oscillations of the contact forces, the element Q2V10P10, using 10 parameters for discretization of velocity and momentum, respectively, provides a relatively smooth response. The figure on the right hand side demonstrates the attractive features of variational mass scaling. In the three-dimensional analysis of the forced vibration of a cantilever beam using 8-node hexahedra, explicit time integration with only 719 steps using a variationally scaled mass matrix perfectly matches the reference solution using lumped mass (LMM), where the size of the critical time step necessitates an analysis with 12900 time steps.

REFERENCES

[1] H. Khenous, P. Laborde, Y. Renard. Mass redistribution method for finite element contact problems in elastodynamics, *European Journal of Mechanics - A/Solid*; 27, 918-932, 2008.

[2] L. Olovsson, M. Unosson, K. Simonsson, Selective mass scaling for thin-walled structures modelled with tri-linear solid elements, *Comp. Mech.* 34,134-136, 2004.

[3] A. Tkachuk and M. Bischoff, Variational methods for selective mass scaling. *Computational Mechanics* 52, pp. 563-570, 2013.

[4] A. Tkachuk, B. Wohlmuth and M. Bischoff, Hybrid-mixed discretization of elasto-dynamic contact problems using consistent singular mass matrices, *International Journal for Numerical Methods in Engineering* 94, pp. 473-493, 2013.