11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

DISCONTINUOUS PETROV–GALERKIN METHOD WITH OPTIMAL TEST FUNCTIONS. PROGRESS REPORT

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Key words: *Minimum residual methods, optimal test functions*

Past June marked the 4th anniversary of the first two ICES Reports [1] in which Jay Gopalakrishnan and I presented a new FE method that automatically guarantees discrete stability by means of a Petrov-Galerkin scheme with optimal test functions computed on a fly. The main idea is very simple: compute (approximately) and use test functions that realize the supremum in the inf-sup condition - the best test functions you can have. Surprise or not, we arrive at a minimum residual method (generalized least squares) in which the approximate solution delivers the best approximation error in a special "energy" (residual) norm.

20+ papers and three Ph.D. dissertations later (see [2] a recent review, and workshop presentations at `https://sites.google.com/site/workshoplmr`), we have managed to establish a fairly complete theory and understanding of the methodology, its potentials and limitations. The presentation will consist of three parts. I will begin with a short review of the main technology focusing on the application aspects: ultraweak variational formulation, computation of element matrices and evaluation of residual. I will flash then a number of representative examples for wave propagation, compressible and incompressible Navier-Stokes from Jamie's, Jesse's and Nate's dissertations. We shall focus on issues of robustness (uniform stability in perturbation parameter) and stability in the coarse mesh regime. I will finish with a short discussion of the technology in context of polygonal elements of arbitrary shape.

REFERENCES

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