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A FIRST ORDER CONSERVATION LAW FRAMEWORK FOR SOLID DYNAMICS

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The formulation of solid dynamics as a set of first order conservation laws with involutions has been developed by the authors in a series of previous publications [1-4]. The use of this formulation overcomes a number of well known locking difficulties experienced by the displacement based approach when low order elements are used. It leads to the same order of convergence for the stress field as for the displacement and enables the extension of a variety of standard CFD techniques to solid dynamics.

The paper will present recently developed extensions of the above concepts for materials where the elastic component of the constitutive relation satisfies polyconvexity, that is is convex in the deformation gradient tensor, its adjoint and its determinant. It will be shown that in such cases, it is possible to derive conservation laws for the extended set of geometric variables and define a convex generalised entropy function [5]. This makes it possible to obtain entropy conjugate variables and symmetrise the resulting system of quasi-linear hyperbolic equations [6]. It will be shown the symmetrised equations are evolution laws for a certain set of stress measures.

The paper will review a number of discretisation techniques for the resulting system of equations, such as Petrov-Galerkin stabilised finite elements and vertex centred finite volume methods. A number of examples in the large strain elastic and elastoplastic regimes, with and without thermal effects will be used to demonstrate the validity of the formulation proposed and the advantages over standard displacement based formulations.

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