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CONVERGENCE PROPERTIES OF THE HIERARCHICAL MODELS IN COUPLED ELECTRO-MECHANICAL PROBLEMS

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The performed research constitutes a part of a general project on adaptive hierarchical modelling and adaptive finite element analysis of coupled electro-mechanical systems composed of elastic structural members and piezoelectric transducers or sensors.

The paper presents results of the numerical, and also theoretical (if available), research on the convergence of the solutions of the model coupled electro-mechanical problems. The problems concern electro-mechanical piezoelectric systems and the systems composed of elastic and piezoelectric members as well. The models applied within the piezoelectric and elastic-piezoelectric structures are of hierarchical character and differ with the constraints imposed on the electric and mechanical fields in the third local direction. In the case of the displacement field, this direction corresponds to the transverse direction within the thin-walled elastic and piezoelectric members. In the case of the electric potential field, this direction coincides with the polarization direction of the piezoelectric parts of the structures. In the case of thin-walled piezoelectric members the third local direction for both the fields is the same. For both fields the highest models of the hierarchies of the mechanical, electric and piezoelectric models correspond to the three-dimensional elasticity and piezoelectricity.

The obtained results concern the h-, p- and q-convergence curves of the purely elastic model problems. Here, h represents an averaged dimension of a finite element, while p and q denote the longitudinal and transverse approximation orders of the three-dimensional displacement field within a finite element. Note that q is equivalent to the order of the hierarchical model applied within the field of displacements. In this paper we also consider the purely electric model problems. The h-, π- and ρ-convergence curves are presented for such models, where π and ρ are the longitudinal and transverse approximation orders of the scalar electric potential field, with ρ equivalent to the order of the hierarchical electric model. Then the h-, p-, q-, π- and ρ-convergence studies are performed for the corresponding coupled electromechanical model problems. The obtained numerical results are compared to the corresponding results of the purely mechanical or electric model problems. All the results are discussed and then compared to the theoretical ones, when available.

The performed research is a preliminary stage for the introduction of the 3D-based hierarchical numerical models for the adaptive modelling and adaptive analysis of the coupled electro-mechanical systems. The mentioned hierarchical numerical models are composed of the 3D-based mechanical, electric and piezoelectric models and the corresponding hierarchical approximations applied within these models. The 3D-based models are equipped with the three-dimensional degree of freedom within the mechanical and electric fields.

In the case of the purely mechanical systems or the displacement field of the piezoelectric models we include the 3D-based first-order shell model, the 3D-based hierarchical shell models conforming to the higher-order shell theories, and the 3D elasticity model as well [1]. Also the 3D-based solid-to-shell or shell-to-shell transition models are employed when necessary. The 3D-based purely electric models, and the electric potential field within the piezoelectric models, include the hierarchy of models constrained with a varying-order polynomial functions in the direction of the electric polarization. Such constraints are introduced in analogy to the polynomial constraints in the transverse direction of the 3D-based shell models. Note that in the case of the piezoelectric models, any combination of the purely mechanical and purely electric models is possible.

In the case of the 3D-based first-order shell model we apply the two-dimensional hierarchical *hp*-approximation, while in the case of the 3D-based hierarchical shell models *hpq*-approximations are utilized [1]. Such approximations are composed of the two-dimensional (in-plane) *hp*-approximation and one-dimensional *q*-approximation in the transverse direction. In the case when 3D-elasticity is applied the thin-walled structures the applied hierarchical approximation is of *hpp* character (the one-dimensional transverse approximation order is equal to the two-dimensional longitudinal one). In the solid structures or such parts of the complex structures one may apply the three-dimensional *h* and *p* approximations of the 3D-elasticty model. In the case of the solid-to-shell or shell-to-shell models the hierarchical approximations are of mixed character, i.e. *hpp/hp* and *hpq/hp* are employed, respectively. As far as the 3D-based hierarchy of electric models is concerned, one applies $h\pi\rho$-approximations which are composed of the two-dimensional $h\pi$-approximation and one-dimensional ρ-approximation in the polarization direction. In the case of the piezoelectric models the mentioned above approximations are applied accordingly.

The obtained convergence results will serve as a hint for elaboration of the error estimation techniques. These results will serve also the elaboration of the effective adaptation algorithms for both the electric and mechanical fields. The estimated values of the modelling, approximation and total errors will be employed for the control of model and discretization adaptivity. The components of the mentioned errors will be defined in the strain energy, electric energy and coupling energy of the system. The starting point for the elaboration of the error estimation techniques will be the equilibrated residual method, while in the case of the adaptive algorithms we will extend the Texas three-step strategy. Both these techniques were applied by us successfully in the case of the purely elastic complex structures [2].

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