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A Semi-Continuous Formulation for Goal-Oriented Reduced-Order Models

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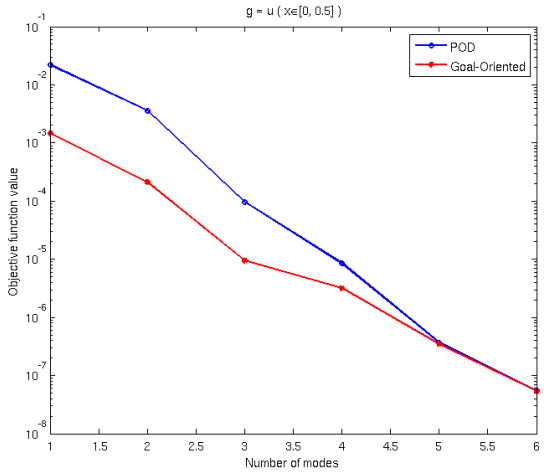
ABSTRACT

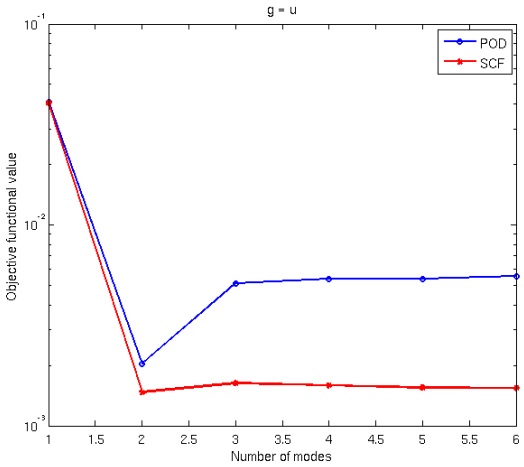
The Proper Orthogonal Decomposition (POD) has been widely used to generate the basis functions for reduced-order models (ROMs). It is optimal in the approximation of reference data in the L_2 norm sense. Its limitations, however, arise from the fact that the POD is purely data-driven and no explicit reference is made to the ROM, which implies that the POD may not be optimal for ROM outputs.

To overcome these drawbacks, Bui-Thanh proposed a ROM-constrained optimisation method which generates basis functions which are optimal for arbitrary ROM outputs. As it was constructed for Linear Time-Invariant (LTI) systems with a fully-discrete character, we refer to it as "fully-discrete formulation (FDF)". The formulation requires system matrices from a discrete description of the dynamics of the data set. Such matrices are not always available, particularly when the reference data comes from an experiment. Furthermore the extension of the FDF to nonlinear PDE and nonlinear ROM outputs is not immediately obvious.

In this contribution, we consider Goal-Oriented, ROM-constrained optimisation in a continuous setting. The reference data, on the other hand, can still be discrete. This approach, which refer to as the Semi-Continuous Formulation (SCF), removes the ambiguity associated with the definition of data system matrices and clarifies the treatment of nonlinear PDE and ROM outputs. We illustrate the use of the SCF for both linear and nonlinear cases, and compare it with the FDF. We also give numerical results and compare them with outputs from classical POD ROMs. Finally, in order to simulate the approximation of experimental reference data, we also consider the optimisation of ROMs based on simplified model equations.



(a) 1D convection diffusion ROM constraint



(b) Approximate ROM constraint

Figure 1: Comparison objective function values from Goal-Oriented SCF and a classical POD ROMs

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