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# CONTINUATION OF EQUILIBRIA AND STABILITY OF NATURALLY CURVED ELASTIC RODS USING AN ASYMPTOTIC NUMERICAL METHOD

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We present a novel simulation technique, based on an asymptotic numerical method, to characterize and quantify the equilibria of naturally curved elastic rods under large geometrically nonlinear displacements and rotations [1]. The 3D kinematics of the assumed inextensible rod is treated in a geometrically exact way by parameterizing the position of the centerline and making use of quaternions to represent the orientation of the material frame [2]. Inextensibility is ensured thanks to Lagrange multipliers. The equilibrium equations are derived from the constrained mechanical energy which takes into account the contributions due to internal moments (bending and twist), external forces and torques. Our use of quaternions allows for the equilibrium equations to be written in a quadratic form and solved efficiently with the asymptotic numerical continuation method introduced in [3]. Based on the asymptotic expansion of the unknown kinematics, Lagrange multipliers and control parameter in a path-parameter variable, this method gives access to analytical equilibrium branches, a.k.a bifurcation diagrams. This is in contrast with the individual solution points attained by classic energy minimization or predictor-corrector techniques. Finally, the stability of the equilibrium is assessed by analyzing the Lyapunov exponents of the Hessian of the projected Lagrangian along the branches. This original numerical method is implemented in the open source software MANLAB: a user-friendly, interactive and object-oriented Matlab path-following and bifurcation analysis program [4].

By way of example, we investigate how the geometrically nonlinear configurations obtained from the writhing of a heavy elastic rod are influenced by its intrinsic natural curvature. To this end, we perform a combination of numerics and precision model experiments on the compression or twisting of a thin rod. Excellent quantitative agreement is found

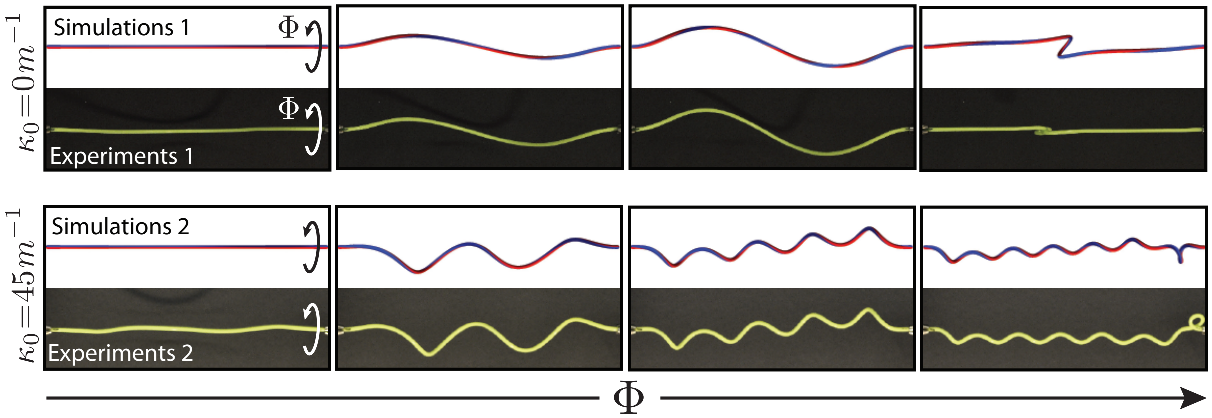


Figure 1: Top views of experimental (black background) and numerical (white background) equilibrium configurations of a heavy slender elastic rod for quasi-statically increasing values of the end-to-end twisting rotation angle $\Phi$. Our numerical model correctly predicts the complex evolutions of equilibrium states of the rod until emergence of a localized state (plectoneme). (Top) Experiment 1 was performed for a naturally straight elastic rod with a natural curvature $\kappa_0 = 0$ m$^{-1}$. (Bottom) Experiment 2 shows a naturally curved elastic rod with a natural curvature $\kappa_0 = 45$ m$^{-1}$.

between experiments and simulations for the underlying buckling instabilities and the characterization of the resulting complex configurations (as illustrated in Fig.1 for the twisting case). We uncover the original effect that weight delays the effect of natural curvature, in a mechanism that results from the complex interplay between constrained geometry, elasticity and weight.

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