11th. World Congress on Computational Mechanics (WCCM XI)
5th. European Conference on Computational Mechanics (ECCM V)
6th. European Conference on Computational Fluid Dynamics (ECFD VI)
July 20 - 25, 2014, Barcelona, Spain

A FULLY-SEPARATED PGD ALGORITHM FOR NONLINEAR PROBLEMS

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Key words: *PGD, Model Reduction, Nonlinear, Separated representation.*

This work is concerned with the solution of nonlinear problems using the Proper Generalized Decomposition (PGD) method [1]. From the weighted residual form of some linear model, PGD uses a Greedy algorithm to construct a reduced basis for the solution. Consider for instance a d-dimensional problem, with $d \in \mathbb{N}^+$, and assume the solution admits a separated representation. Then, the d-dimensional problem that has to be solved to compute each basis function can be decoupled, in the best case, into a set of d 1-dimensional problems. All of them are related one to another through some "coupling coefficients" coming from a series of 1-dimensional integrals. Because solving 1-dimensional problems is much less expensive than solving a single d-dimensional problem, PGD is not only seen as a multilinear solver but also as a Model Order Reduction (MOR) method.

If a nonlinear problem wants to be addressed using PGD, a linearization scheme has to be put on top of it, and thus PGD will solve a sequence of linearized equations until the convergence is reached. Observe that:

1. Linearized equations involve, at some point, the evaluation of the nonlinear term.
2. Both the linearization scheme and the construction of the reduced basis are iterative procedures taking place simultaneously.

Concerning the first issue, as in general no separated representation of the nonlinear term is known, the d-dimensional problem cannot be easily decoupled into d 1-dimensional problems. Or in other words, the "coupling coefficients" can only be obtained at the price of computing d-dimensional integrals. In consequence, the computational complexity is not effectively reduced as it was in the linear case.

This work explores different alternatives to reduce the computational complexity of the nonlinear term. In first place, interpolation techniques such as Empirical Interpolation Method (EIM) [2], or its discrete counterpart (DEIM) [3], are considered. These methods perform a sampling to construct a projection basis for the nonlinear term. Once this basis is computed, the nonlinear term is interpolated using the "magic points" algorithm [4]. This method is well adapted to *a posteriori* MOR methods, but for *a priori* methods such as PGD, the election of the projection basis is not evident [5]. For this reason, rather than interpolation, this work proposes a technique that allows computing a separated approximation of the nonlinear term, and thus, offers an effective reduction of the computational complexity.

From the second issue, it follows that the rank of the computed approximation is, in general, subsidiary of the convergence of the nonlinear scheme. In particular, suppose that the basis functions that solve the k-th linearization of the problem have already been computed. If at the k-th iteration, the solution is still far from convergence, there is no particular reason to think that the converged solution could be written in terms of those basis functions. In consequence, depending on the manner the reduced basis is enriched we might add terms unnecessarily just because at a given iteration of the nonlinear scheme the solution is still far from convergence.

This work analyzes the impact of the nonlinear convergence on the rank of the computed approximation, and proposes some alternatives to circunvent this dependency.

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