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SOLVING HIGHER ORDER BOUNDARY VALUE PROBLEM CONTAINING UNKNOWN PARAMETERS

Renata Filipowska

PhD. Renata Filipowska, Institute of Applied Informatics, Faculty of Mechanical Engineering, Cracow University of Technology, renata.filipowska@op.pl

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Many engineering problems can be described by boundary value problem. Usually it consists of a non-linear differential equation and boundary conditions, which number corresponds to the order of this equation. There are a few methods of solving this problem, e.g. shooting method. In order to use this method, it is necessary to convert the boundary value problem to the appropriate initial value problem. In the non-standard case, the number of boundary conditions may exceed the order of the differential equation. In this case the differential equation must contain the appropriate number of unknown parameters. To solve this kind of problems a modification of the shooting method, presented in the present paper, can be applied. This method is based on the so-called sensitivity functions and enables to find the values of these parameters that make the associated initial value problem fulfill all boundary conditions with the prescribed accuracy.

Let us consider the fourth-order differential equation, which contains the unknown p parameter

$$y''''(x,p) = f\left(x, y(x,p), y'(x,p), y''(x,p), y'''(x,p), p\right) \tag{1}$$

and the five boundary conditions

$$y(a) = y_a\,,\; y''(a) = y_{ba}\,,\; y'''(a) = y_{ta}\,, y(b) = y_b\,, y'(b) = y_{pb} \tag{2}$$

In order to use the shooting method, it is necessary to convert the boundary value problem to the initial value problem consisting of the equivalent system of four first-order differential equations and the initial conditions, which contain the second unknown q parameter

$$y(a) = y_a\,, y'(a) = q\,,\; y''(a) = y_{ba}\,,\; y'''(a) = y_{ta} \tag{3}$$

To apply the method presented, one needs to solve the initial value problem with appropriately changing values of parameters p and q.

To this end we need to expand the functions, which the finish conditions (2) are given for, into Taylor's series around the trial values of p and q parameters. With the aid of these expansions the unknown sensitivity functions are defined. In the case of the problem described by (1)-(2), differentiating the system of four first-order differential equations with respect to *p* and *q* parameters, one obtains an additional system of eight equations and appropriate initial conditions. Finally, in each iteration initial value problem consisting of the twelve first-order differential equations has to be solved. Calculations are carried out until the solution fulfills boundary conditions (2) at the right end of the interval.

In the present work the method has been applied to solve a few non-standard boundary value problems consisting of fourth-order differential equations. It should be noted that the method is of the second order accuracy, what is also proved in the paper.

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