

Regularizing Nonlinear Systems with Discontinuous Solutions in Higher Order Methods

Craig Michoski¹, Clint Dawson¹, Dam Wirasaet³, Ethan Kubatko², Joannes Westerink³

¹ University of Texas ICES, Austin, TX 78712, michoski@ices.utexas.edu d URL

² The Ohio State University, Columbus, OH 43210, ekubatko@gmail.com

³ Notre Dame University, Notre Dame, IN 46556, dwirasae@nd.edu

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Nonlinear systems of equations often demonstrate complicated regularity features that can be obfuscated by diffuse numerical methods, or methods that preserve sub-scale features that are incommensurate with the signature behavior of the fully coupled system. Using a discontinuous Galerkin finite element method, we study a nonlinear system of advection-diffusion-reaction equations and aspects of its regularity and stability. For numerical analysis, we present a family of regularized solutions consisting of: 1) a sharp, computationally efficient slope limiter, known as the BDS limiter¹, 2) a standard spectral filter², and 3) a novel artificial diffusion algorithm (motivated from entropy viscosity type methods) with a solution-dependent entropy sensor. We analyze these three numerical regularization methods on a classical numerical test in order to tease out the strengths and weaknesses of each, and to determine the circumstances where each is most/least appropriately applied. We then test these methods on a very large application model, to see how they perform.

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