11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

IMPROVED ACCURACY OF HIGH-ORDER WENO FINITE VOLUME METHODS ON CARTESIAN GRIDS

Pawel Buchmueller*, Christiane Helzel †

*Ruhr-University Bochum, Department of Mathematics,
Universitaetsstrae 150, 44801 Bochum,
pawel.buchmueller@rub.de

† Ruhr-University Bochum, Department of Mathematics,
Universitaetsstrae 150, 44801 Bochum,
christiane.helzel@rub.de

Key words: *Weighted essentially non-oscillatory (WENO) schemes · Finite volume methods · high-order methods · Euler equations · MHD · hyperbolic PDEs.*

We will present our recent result on the construction of high order WENO finite volume methods for the approximation of hyperbolic partial differential equations on Cartesian grids. The simplest way to use WENO methods on multidimensional Cartesian grids consists in applying a one-dimensional WENO scheme in each direction. This spatial discretization is typically combined with a Runge-Kutta method in time. In the finite difference context, such WENO methods lead to a high order accurate approximation. In contrast to this, finite volume WENO methods based on a dimension-by-dimension approach are only second order accurate for nonlinear problems, see [2].

The loss of accuracy can be avoided if high order quadrature rules are used for the flux computation. This requires some form of a multidimensional reconstruction as described in [5] for general grids or in [3] for Cartesian grids. However, such high order WENO finite volume methods are computationally much more expensive than the simple dimension-by-dimension approach.

Our modified dimension-by-dimension approach uses one-dimensional WENO reconstruction and requires only one flux computation per grid cell interface. The crucial step to avoid the loss of accuracy is a transformation between point values and averaged values of the physical quantities at several places of the algorithm. Such a transformation has also been used in the recently proposed fourth order accurate finite volume method of McCorquodale and Colella [4].

Several test computations, including Euler equations of gas dynamics and ideal magneto-hydrodynamics will be discussed.

REFERENCES

[1] P. Buchmueller and C. Helzel. Improved Accuracy of High-Order WENO Finite Volume Methods on Cartesian Grids. *Submitted to J. of Sci. Comput.* 2013.

[2] R. Zhang, M. Zhang, and C.-W. Shu. On the order of accuracy and numerical performance of two classes of finite volume WENO schemes. *Commun. Comput. Phys.*, 9:807–827, 2011.

[3] J. Casper and H.L. Atkins. A finite-volume high-order ENO scheme for two-dimensional hyperbolic systems. J. Comput. Phys., 106:6276, 1993. J. Comput. Phys.,106:6276, 1993.

[4] P. McCorquodale and P. Colella. A high–order finite volume method for conservation laws on logically refined grids. *Commun. App. Math. and Comp. Sci.*, 6:1–25, 2011.

[5] P. Tsoutsanis, V.A. Titarev, and D. Drikakis. Weno schemes on arbitrary mixedelement unstructured meshes in three space dimension. *J. Comput. Phys.*, 230:1585-1601, 2011.