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THREE DIMENSION LOCALIZED MULTIGRID CRACK SIMULATION WITH DIRECT ESTIMATION OF STRESS INTENSITY FACTORS

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The considered problem is the simulation of a cracked body under Linear Elastic Fracture Mechanics (LEFM) hypotheses. This problem presents a stress/strain singularity at the crack tip. The proposed simulation method, based on crack tip analytical solution, provides directly an evaluation of the stress intensity factors (SIF), which quantify the crack singularity.

A closed-form solution, well fitted to describe the mechanical behavior at crack tip, is used as mechanical basis in the crack tip vicinity. This solution arises from an asymptotic study of a plane crack in a semi infinite linear elastic medium under strain conditions. This study provides an asymptotic series expansion referred as Williams series [1]. The three dimensional asymptotic expansion is not available, thus the two dimensional modes are extended. To do so, the three modes of rupture are considered (I, II in-plane and III out-of-plane) and their evolution along the front is assumed continuous and discretized in a finite element sense

$$\boldsymbol{u}_W(\boldsymbol{X}) = \boldsymbol{u}_W(r,\theta,s) = \sum_s \left[\sum_{n=0}^{n_{max}=\infty} \sum_{i=I,II,III} (b_i^n)^s \, \boldsymbol{f}_i^n(\theta) \cdot r^{\frac{n}{2}} \right] \varphi^s(s). \tag{1}$$

The simulation of the whole problem is performed using a domain decomposition method. This method, proposed in [2] for 2D simulations, is illustrated figure 1. It consists in using the eXtended Finite Element Method (X-FEM) to account for the structural behavior in Ω_X and an analytical patch in the crack tip vicinity Ω_W. At their interface Γ_W, the mechanical fields are incompatible. Displacement compatibility is enforced in a weak sense by Lagrange multipliers $\boldsymbol{\lambda}$

$$\langle \boldsymbol{u}_X - \boldsymbol{u}_W, \boldsymbol{\lambda} \rangle_{\Gamma_W} = \int_{\Gamma_W} \boldsymbol{\lambda} \cdot (\boldsymbol{u}_X - \boldsymbol{u}_W) \; dS = 0. \tag{2}$$

The displacement is enriched with the singularity in the analytical patch. This domain is link to the non enriched domain Ω_W on the interface Γ_W, thus there is no half enriched element known as blending-element.

In the analytical patch, the equilibrium problem is solved with a Galerkin method for which mechanical fields are discretized on a truncation of the asymptotic expansion (1). Such a discretization directly provides the SIF evolution along the crack front: $K_i(s) = \sum_s \left(b_i^1\right)^s \varphi^s(s)$, for $i = I,II$ or III. It worth noticing that some other meaningful asymptotic coefficients are provided, such as the T-stress (order $n = 2$).

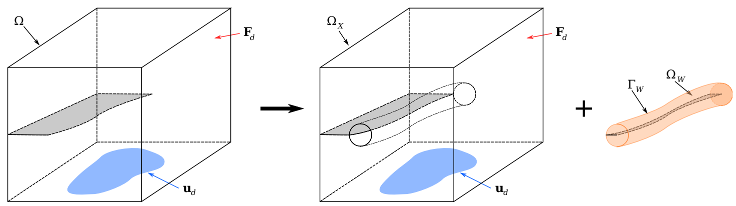

Figure 1: Domain decomposition between an analytical patch Ω_W and an X-FEM domain Ω_X.

The Williams series expansion in crack tip vicinity is only valid for straight cracks, thus the analytical patch must be small enough to be localized in an area where the crack curvature can be neglected. Two dimension simulations have shown [2] that the radius of the patch have to be larger than the mesh size of the neighboring X-FEM grid. Thus local refinement is performed with an X-FEM localized multigrid method as introduced by [3]. The analytical patch is embedded in the finest grid only and the coarse grids are based on X-FEM frameworks.

The proposed method evaluates directly the evolution of asymptotic coefficients (*i.e.* K_i and T-stress) along the crack front in LEFM situations. Three dimensional mixed-modes problem can be considered. The underlying X-FEM discretization is made as fine as needed thanks to a localized multigrid method. The crack tip singularity is well represented with dedicated fields.

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