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NON-DETERMINISTIC SIMULATIONS WITH CFD ROBUSTNESS PROPERTIES

Jeroen A.S. Witteveen*[1] and Gianluca Iaccarino[2]

[1] Center for Mathematics and Computer Science (CWI), Science Park 123, 1098XG Amsterdam, The Netherlands, jeroen.witteveen@cwi.nl, http://www.jeroenwitteveen.com
[2] Mechanical Engineering, Stanford University, Building 500, Stanford, CA 94305-3035, USA, jops@stanford.edu, http://www.stanford.edu/~jops

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Non-intrusive uncertainty quantification (UQ) is applicable to many branches of the computational sciences and it has a strong foundation in mathematical approximation and interpolation theories. The unique contribution that computational fluid dynamics (CFD) can bring to the field of numerical methods for computationally intensive stochastic problems is the robust approximation of discontinuities in the probability space. There is extensive experience in the finite volume method (FVM) community with original robustness concepts for the reliable solution of discontinuities in the form of shock waves and contact surfaces in flow fields. We extend the essentially non-oscillatory (ENO) stencil selection and the subcell resolution (SR) approach here to the probability space. They are introduced into the simplex stochastic collocation (SSC) method for UQ. Results for Sod's Riemann problem in Figures 1-3 and for the RAE2822 airfoil with n=5 samples in Figure 4 demonstrate their effectiveness in non-deterministic CFD.

The ENO spatial discretization [1] achieves an essentially non-oscillatory approximation of the solution of hyperbolic conservation laws. The notion of subcell resolution in FVM originated also from Harten [2] to prevent the smearing of contact discontinuities in the solution of hyperbolic conservation laws in the physical space X.

The SSC method [3,4] is an advanced adaptive multi-element UQ method based on a simplex tessellation of the probability space Ξ with sampling points $\boldsymbol{\xi}_k$ at the vertexes of the simplex elements Ξ_j. The polynomial approximation in Ξ_j is built using higher degree interpolation stencils S_j, with local polynomial degree p_j, consisting of the sampling points $\boldsymbol{\xi}_k$ in the vertexes of surrounding simplexes. The degree p_j is controlled by a local extremum conserving (LEC) limiter, which reduces p_j and the stencil size to avoid overshoots in the interpolation of the samples where necessary. SSC employs adaptive refinement measures based on the hierarchical surplus and the geometrical properties of the simplexes to identify the location of discontinuities.

In order to obtain a more accurate solution of nonlinear response surfaces, ENO-type stencil selection is introduced into the SSC–ENO method [5]. For each Ξ_j, r_j candidate stencils $S_{j,i}$, i=1,…,r_j, are constructed that contain Ξ_j, and the stencil S_j is selected that results in the smoothest interpolation $w_j(\boldsymbol{\xi})$ in terms of the highest $p_{j,i}$.

However, in problems where the location of a discontinuity in the physical space is random, adaptive refinement in the probability space proves ineffective. For each point $\boldsymbol{x}$ in the physical space, the spatial discontinuity namely results in a jump at a different location $\boldsymbol{\xi}$ in the probability space.

Therefore, we introduce the concept of subcell resolution into the SSC–SR method [6] by extracting the discontinuity location $\boldsymbol{x}_{\text{disc}}$ in the physical space from each of the deterministic simulations for the sampled random parameter values ξ_k. These physical discontinuity locations $\boldsymbol{x}_{\text{disc}}$ are interpolated in the stochastic dimensions to derive a relation for the location of the discontinuity ξ_{disc} in the probability space as a function of the spatial coordinate $\boldsymbol{x}$. In the discontinuous cells, the interpolations $w_j(\xi)$ of the neighboring cells Ξ_j are then extended from both sides up to the predicted discontinuity location ξ_{disc}. This leads to a genuinely discontinuous representation of the jump in the interior of the cells in the probability space.

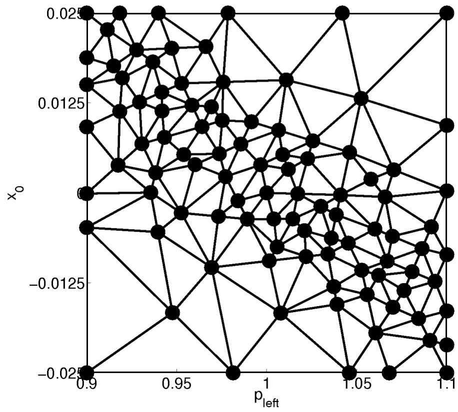

Figure 1: Simplex stochastic collocation.

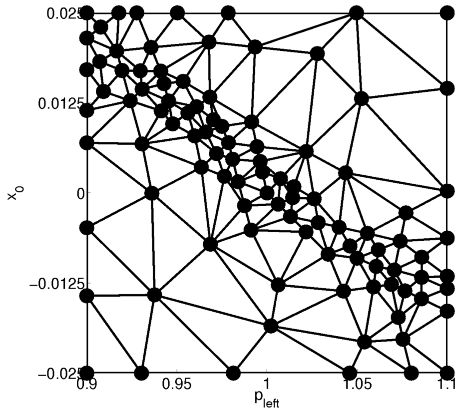

Figure 2: ENO stencil selection.

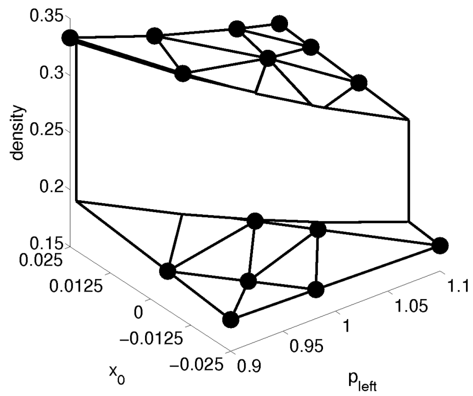

Figure 3: Subcell resolution.

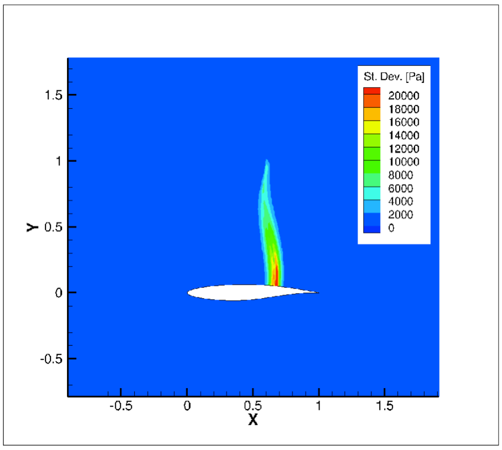

Figure 4: Pressure standard deviation.

REFERENCES

[1] A. Harten, S. Osher, Uniformly high-order accurate nonoscillatory schemes I, *SIAM Journal on Numerical Analysis*, Vol. **24**, pp. 279–309, 1987.

[2] A. Harten, ENO schemes with subcell resolution, *Journal of Computational Physics*, Vol. **83**, pp. 148–184, 1989.

[3] J.A.S. Witteveen, G. Iaccarino, Simplex stochastic collocation with random sampling and extrapolation for nonhypercube probability spaces, *SIAM Journal on Scientific Computing*, Vol. **34**, pp. A814–A838, 2012.

[4] J.A.S. Witteveen, G. Iaccarino, Refinement criteria for simplex stochastic collocation with local extremum diminishing robustness, *SIAM Journal on Scientific Computing*, Vol. **34**, pp. A1522–A1543, 2012.

[5] J.A.S. Witteveen, G. Iaccarino, Simplex stochastic collocation with ENO-type stencil selection for robust uncertainty quantification, *Journal of Computational Physics*, Vol. **239**, pp. 1–21, 2013.

[6] J.A.S. Witteveen, G. Iaccarino, Subcell resolution in simplex stochastic collocation for spatial discontinuities, *Journal of Computational Physics*, Vol. **251**, pp. 17–52, 2013.