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A MIXED MULTISCALE MODEL ACCOUNTING FOR THE CROSS TERM OF THE SUB-GRID SCALE STRESS

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Most approaches for modeling the sub-grid scale (SGS) stress, in large-eddy simulation (LES) of turbulent flows, are based on a purely dissipative formulation. Typically, the SGS stress $\widetilde{T}_{ij} = \widetilde{u_i u_j} - \widetilde{u}_i \widetilde{u}_j$ is modeled as $\widetilde{T}_{ij}^M = -2\nu_e \widetilde{S}_{ij}$, with $\widetilde{S}_{ij}$ the resolved strain rate and with a closure equation for the SGS eddy viscosity ν_e. The simplest model is that by Smagorinsky [1]: $\nu_e = C\Delta^2|\widetilde{S}|$ with Δ, the local effective grid size. A dynamic version, to obtain dynamically the C coefficient, was proposed by Germano et al. [2]. A model aiming at discriminating between the scales of the flows was introduced by Hughes et al., with the so-called "variational multiscale" (VMS) model [3]: a sharp Fourier cut-off was used to discriminate between the "large" and "small" scales of the LES field, and the SGS dissipation was restrained to the small scales. Regularized versions were later proposed by others; also the "regularized VMS" (RVMS) model by Jeanmart and Winckelmans [4], using high order high-pass filters that are efficiently obtained in physical space, by using iterations. These models have the form $\widetilde{T}_{ij}^M = -2\nu_e \widetilde{S}_{ij}^s$, where the small scales field, $\widetilde{u}_i^s$, is obtained by the high-pass filtering. Various versions were proposed, as the effective viscosity ν_e can itself be evaluated using either $|\widetilde{S}|$ or $|\widetilde{S}^s|$.

We recall that the SGS stress tensor is in fact made of two parts: the Reynolds term and the cross term: $\widetilde{T}_{ij} = \widetilde{u_i' u_j'} + (\widetilde{\widetilde{u}_i u_j'} + \widetilde{u_i' \widetilde{u}_j}) = \widetilde{R}_{ij} + \widetilde{C}_{ij}$. Considering homogeneous isotropic turbulence (HIT), and computing the exact SGS terms from direct numerical simulation (DNS) fields, it is seen that the dissipation spectra of those terms are very different; yet they both dissipate energy spectrally (i.e., at all wavenumbers), and thus also globally. When considering the physical dissipation in physical space, the distribution of dissipation is then usefully analyzed using pdfs: each of those terms is seen to correspond to a negative dissipation (i.e., to backscatter) at many locations. The different spectral behaviors motivate developing mixed models that mimic the correct effect of the total SGS stress: Reynolds term plus cross term. Such a mixed model was developed by Wang and Oberai [5], in the context of the variational multiscale approach. We here consider the context of LES using classical approaches (high order finite differences or Fourier-based pseudo-spectral methods), and using high order filtering that is efficiently obtained in physical space. Our cross term model is developed based on a scale-similarity argument, with a further modification to obtain Galilean invariance. It makes use of the high order filtering operations, and it indeed produces significant backscatter, while being globally dissipative. It is complemented by a purely dissipative term: either the dynamic RVMS model (then also using the high order filtering) or the dynamic Smagorinsky model. The dynamic procedure is done so as to obtain, at all times, the same total dissipation as that provided by a dynamic Smagorinsky model used solely (as in Park and Mahesh [6]). The mixed model is then tested in decaying HIT, started from a truncated reference DNS. We examine the time evolution of the dissipation (also from of each part of the model) and of the spectrum, and compare them to those of the truncated reference DNS.

We stress that such mixed model should not be confused with the mixed models that were developed for LES with regular (explicit or implicit) filtering considered in addition to the cut-off filter by the LES grid, and thus producing both sub-grid scales (SGS) and sub-filter scales (SFS) stresses; such as the mixed model by Winckelmans et al. [7].

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