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AN OPTIMAL ADAPTIVE FEM FOR EIGENVALUE CLUSTERS

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The adaptive finite element approximation of multiple eigenvalues of the model problem $-\triangle u = \lambda u$ leads to the situation of eigenvalue clusters because the eigenvalues of interest and their multiplicities may not be resolved by the initial mesh. The optimality analysis of adaptive finite element methods in the literature is based on the comparison of the finite element solutions on different meshes. In the case of multiple eigenvalues, this leads to the difficulty that the discrete orthonormal systems of eigenfunctions produced by the adaptive algorithm may change in each step of the adaptive loop. In practice, little perturbations in coefficients or in the geometry immediately lead to an eigenvalue cluster (described by an index set $J = \{n+1, \dots, n+N\}$). This talk presents an adaptive algorithm for eigenvalue clusters. The main result shows that the the error quantities

$$\left(\frac{|\lambda_k - \lambda_{\ell,k}|}{\lambda_{\ell,k}}\right)^{1/2} \quad \text{and} \quad \sup_{j\in J} \sup_{\substack{w\in E(\lambda_j)\\ \|w\|=1}} \inf_{v_\ell \in W_\ell} \|\!|w - v_\ell|\!\|$$

decay at optimal rate in terms of degrees of freedom. One subtle aspect is the dependence of the parameters on the smallness of the initial mesh and the initial resolution of the cluster and its length.

Numerical tests suggest that the adaptive cluster approximation seems to be superior compared to the use of an adaptive scheme for each eigenvalue separately, even if the multiplicies are known.