

MODELLING NON-QUADRATIC ANISOTROPIC YIELD CRITERIA AT FINITE STRAINS WITH MIXED ISOTROPIC-NONLINEAR KINEMATIC HARDENING: APPLICATION TO SHEET METAL FORMING

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Nowadays, seeking for structures with higher performance and lower weight, industry has increased the use of improved metallic alloys, such as advanced high strength steels, which on the other hand present complex anisotropic behaviours. Due to the strong implications that such anisotropic behaviours have in practical applications, e.g. in sheet metal forming, significant attention to the subject of anisotropic plasticity at finite strains have been given to in the last decade. The present work focus on the development and the numerical implementation in Finite Element codes of a general finite strain material model with mixed isotropic-nonlinear kinematic hardening, that accounts for advanced and complex non-quadratic anisotropic yield criteria. The motivation for the present research comes from the fact that the use of both anisotropic hardening laws and advanced anisotropic yield criteria is essential for a reliable prediction of deep drawing and springback results using numerical simulations.

The developed formulation is based on the one firstly proposed by Vladimirov et. al. [1], while in the present work an important step towards generality is carried out, with its extension to any anisotropic yield criteria. The finite strain model is derived from a thermodynamically consistent framework and relies on the multiplicative split of the deformation gradient in the context of hyperelasticity. This multiplicative decomposition is widely accepted and employed with isotropic finite strain plasticity since it is theoretically sound and physically motivated in the framework of slip systems.

The nonlinear kinematic hardening approach is introduced in the constitutive model as a continuum extension of the classical rheological model of Armstrong-Frederick kinematic

hardening, and using only symmetric strain-like internal variables. It is worth noted that the material model is presented within a framework suitable for any advanced (and generic) yield criteria, where detailed information on these complex yield criteria can be found in the work of Banabic et. al. [2].

In order to integrate the evolution equations, allowing for their computational implementation, the exponential map procedure is employed [3], and solved within an implicit framework. This algorithmic strategy has the advantage of preserving both the plastic incompressibility (which is a property of most metallic materials in the plastic range), and the symmetry of the internal variables. Furthermore, since the model is defined only by symmetric variables, the spectral decomposition is introduced in order to evaluate the exponential tensor functions in closed form.

The quality of the results coming from the developed and implemented model is assessed in a number of benchmarks in sheet metal forming, and compared with experimental data and numerical results. In the end, a reliable and robust algorithm and programming framework is provided, suitable for direct implementation in commercial Finite Element codes, such as Abaqus (Simulia) and Marc (MSC-Software) packages, which allows for the simulation of industry-relevant sheet metal forming processes.

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