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Extended-Finite Element Method with 3D quadratic elements: integration and conditioning issues.

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The Extended Finite Element Method (X-FEM) was introduced in [1], for the simulation of cracks or interfaces without re-meshing. The X-FEM introduces a new set of degrees of freedom to the standard displacement field as well as new interpolation functions for these extra degrees of freedom. These functions describe more accurately singularity/discontinuity than standard polynomial functions [3]. The level-set method [2] helps the positioning of those singularities/discontinuities anywhere in the bulk as the crack/interface is defined as the iso-zero of a level set function.

Integration of irregular quantities on enriched elements, requires a special care since, Gaussian quadrature is not optimal for quantities lacking regularity. A common integration strategy is then to split elements into sub-cells over which quantities become regular.

Sub-cells have 2 practical advantages: on one hand, sub-cells are simple sub-domains to handle, although the geometry of the iso-zero might be complex. If sub-cells are generic enough, generic gauss points schemes can be used for integration. On the other hand, sub-cells transform the iso-zero (an implicit equation) into an explicit boundary, which is displayed after post-processing.

Sub-cell strategy is implemented in EDF's software Code_Aster, with 3D quadratic elements used to improve the interpolation accuracy of the iso-zero and the displacement field resolution. With linear elements, the splitting of any given element into sub-cells, is a simple procedure. On the opposite, the splitting of quadratic elements involves positioning middle points on the edges of sub-cells and along the iso-zero. This procedure might be complex particularly in 3D: when computing middle points, the curvature of the elements and of the iso-zero has to be taken into account. However, this improves the overall accuracy of the integration.

Geometric enrichment around the crack tip [3] and closeness between nodes and iso-zero, lead to conditioning issues. To address each conditioning issue, we have adapted specific pre-conditioners [3][4]. In fact, the real concern is the loss of a minimum of information during the assembly process to solve the equilibrium equations as machine precision doesn't allow operations between numbers with great contrast. Hence, a pre-conditioner is proposed to adjust the contrast ratio between integrated quantities on sub-cells which have a huge difference in size.
On one hand, we introduce a “local preconditioning” working as an improvement of the formulation of local enrichment functions to adjust the contrast ratio between integrated

quantities on sub-cells which have a huge difference in size and on the other hand, a “global preconditioning” which comes as an improvement of the solver in case local pre-conditioning is not possible.

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