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# DEFORMATION BANDING IN METAL CRYSTALS AS A MATERIAL INSTABILITY: THEORY, ALGORITHM AND MODELING

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Plastic deformation of metal crystals with multiple active slip-systems exhibits a tendency to develop deformation bands. The phenomenon is widely observed experimentally and is usually explained by easier plastic flow when local simultaneous operation of fewer slip-systems is required. The related hypothesis is that deformation banding occurs since it is energetically preferable to uniform plastic deformation. This paper addresses the question how to transform that qualitative hypothesis into a computational approach based on the concept of instability of a plastic deformation path.

A general energy criterion of material instability in rate-independent solids [1] employs the notion of quasiconvexity loss of the incremental constitutive potential. For ductile single crystals with incremental slip-system shears $\Delta\gamma^\alpha \geq 0$ governed by the Hill-Rice theory, the material instability criterion can be re-formulated in the form [2]

$$\inf \frac{1}{|\mathcal{M}|} \int_{\mathcal{M}} \Delta w(\Delta\mathbf{F}, \Delta\gamma^\alpha)\, \mathrm{d}\mathbf{X} < \Delta w(\Delta\mathbf{F}_0, \Delta\gamma_0^\alpha) \qquad \text{subject to } \mathbf{u} = \mathbf{0} \text{ over } \partial\mathcal{M}, \quad (1)$$

where $\Delta\mathbf{F}(\mathbf{X}) = \Delta\mathbf{F}_0 + \nabla\mathbf{u}(\mathbf{X})$ is a spatially nonuniform increment of the deformation gradient while $(\Delta\mathbf{F}_0, \Delta\gamma_0^\alpha)$ corresponds to uniform deformation. $\Delta w$ expresses the total deformation work increment to *second-order* terms, which is a distinctive feature of the approach. The function $\Delta w$ is specified in [2] in the case when nonlocal and interfacial energy effects are neglected. It is shown that the selective symmetrization of the local interaction matrix between active slip-systems [3] is required to find a correct solution by the incremental energy minimization represented by the left-hand side in (1).

The energy criterion for incipient deformation banding in single crystals follows from (1) in the special case when $\Delta\mathbf{F}$ takes two distinct values in two families of parallel and geometrically compatible deformation bands of unknown orientation and volume fractions.

The algorithm to solve the constrained minimization problem (1) for deformation banding is developed by using the augmented Lagrangian method. An implicit time-integration scheme has been implemented within the *Mathematica* package. The tensor exponential map is used to discretize the equation for the rate of plastic deformation gradient. The set of active slip systems is not known a priori and is allowed to change along a loading path. The search strategy for the actual set of active slip-systems is embedded in a natural way in the incremental energy minimization.

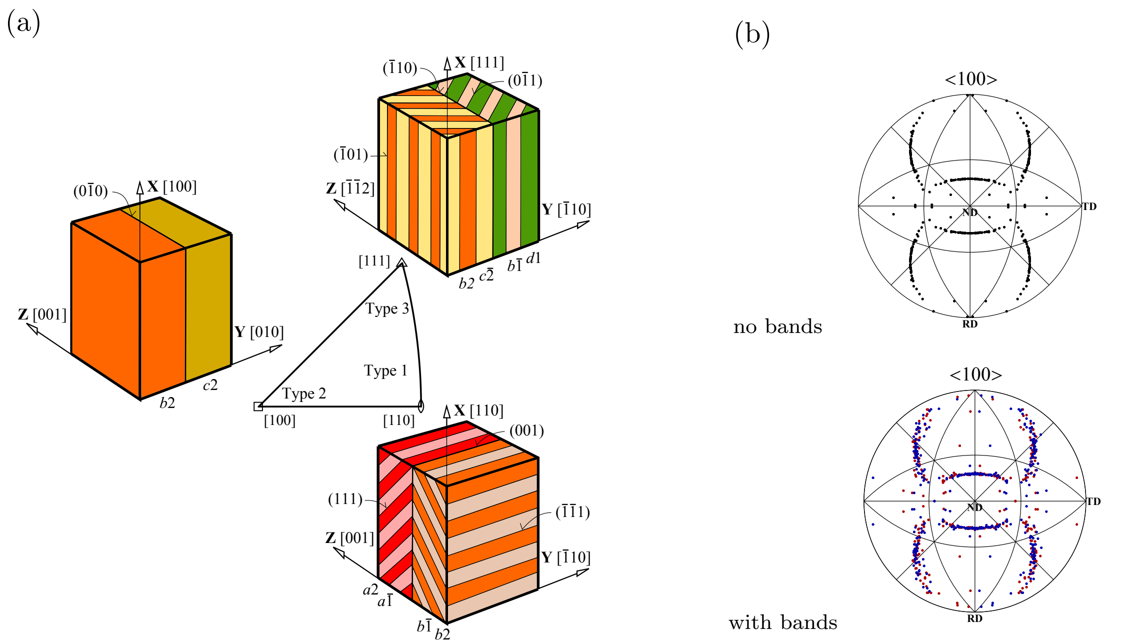


**Figure 1**: Selected examples.

A number of examples have been calculated. Two of them are visualized in the figure and deal with the laminate microstructure formation and evolution within crystalline grains of a Cu polycrystal with the use of the Taylor averaging scheme. Fig. 1(a) illustrates the microstructures in three differently oriented grains in the initial stage of tension, and Fig. 1(b) shows the effect of deformation banding on the texture after large strain due to rolling up to the thickness reduction of 92%. A qualitative agreement with experimental observations is concluded.

## REFERENCES

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