

# Crack growth analysis by a NURBS-based isogeometric boundary element method

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## ABSTRACT

This work outlines an isogeometric boundary element method (IGABEM) to simulate crack growth in a two-dimensional linear-elastic setting. The method is based on the work of [1,2] in which NURBS-based functions are used to approximate both the geometry and analysis fields. This approximation was found to offer significant savings in human intervention through the circumvention of mesh generation and to provide superior accuracy properties over conventional discretisation procedures, in linear elastostatics.

The motivation for this work is the observation that a piece of software, even commercial is yet to be devised which is able to tackle effectively, robustly and accurately the prediction of the tolerance to damage of monolithic industrial structures. The most noteworthy advances in this area are probably within the realm of the extended finite element method [3,4].

In this work, we use the attractive features of IGABEM to construct a tool able to simulate crack growth, directly from CAD, without meshing or remeshing at any stage of the simulation. It is well-known that a straightforward application of the boundary element method to crack problems leads to a singular system as a result of coincident crack surfaces. In this work, we overcome this issue by adopting the dual boundary element method in which independent integral equations are utilised on opposite crack surfaces during collocation [5]. A tradeoff is the increased effort required to integrate hypersingular terms and we outline an algorithm to compute such terms. The crack growth path is described through a NURBS curve using well-established Computer Aided Geometric Design algorithms that allows for smooth continuity between crack growth iterations [6]. To evaluate stress intensity factors, the  $J$ -integral and  $M$ -integral are used and compared against closed-form solutions to verify the accuracy of the implementations. Comparisons are made with Lagrangian discretisations applied to collocation BEM and symmetric Galerkin BEM. Finally, crack propagation paths are verified against experimental results and numerical results obtained through the extended finite element method and a quasi- $C_1$  extended finite element formulation.

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