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A FULLY IMPLICIT LOG-CONFORMATION FORMULATION

Philipp Knechtges*, Stefanie Elgeti and Marek Behr

Chair for Computational Analysis of Technical Systems (CATS)
CCES, RWTH Aachen University, Schinkelstr. 2, 52062 Aachen, Germany
`{knechtges,elgeti,behr}@cats.rwth-aachen.de`
`http://www.cats.rwth-aachen.de`

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The attempt to numerically solve the governing equations of viscoelastic fluids for high Weissenberg numbers often leads to unstable or incorrect results. This so far insufficiently resolved problem has been commonly referred to as the High-Weissenberg Number Problem (HWNP). One of its causes is the inability of numerical schemes to ensure positive definiteness of the conformation tensor at all times; a condition that the exact solution – if existent – always obeys [1]. In order to build a positivity-preserving scheme, Fattal and Kupferman proposed to replace the stress or the conformation tensor in the calculations by a new degree of freedom, the matrix logarithm of the conformation tensor [2]. Now it is ensured by design that the conformation tensor stays positive definite in the approximated setting. This, as well as related types of methods, are categorized as Log-Conformation or just Log-Conf Methods. Special to the construction in [2] is that the new constituitive equation involves an algebraic decomposition of the velocity gradient with respect to the conformation tensor, such that the new set of governing equations immediately acquires an iterative character.

In this presentation we will give a full substitute of the governing equations in terms of the logarithm of the conformation tensor, which does not involve an algebraic decomposition. In addition to offering new opportunities to tackle the HWNP from the analytical side, it allows to construct a Newton-Raphson algorithm that uses the exact analytical expressions for the variational derivatives. Furthermore, we will demonstrate the implementation of the Oldroyd-B model in our finite-element flow solver XNS on the example of the flow around a confined cylinder [3, 4, 5]. In order to stabilize the linear equation system a mixture of SUPG and Galerkin/Least-Squares approaches is used. We will also present the application to problems in mechanical engineering, e.g., die swell behind an extrusion nozzle.

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