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UNCERTAIN MULTIMODE FAILURE AND LIMIT ANALYSIS OF SHELLS

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The plastic collapse limit and the shakedown limit which define the load-carrying capacity of structures are important in assessing the structural integrity. Due to the high expenses of experimental setups and the time consuming full elastic-plastic cyclic loading analysis, the determination of these limits by means of direct plasticity methods has been of great interest to many designers. Moreover, a certain evaluation of structural performance can be conducted only if the uncertainty of the actual load-carrying capacity of the structure is taken into consideration since all resistance and loading variables are random in nature. To ensure the safety of the structures to be designed, two approaches are normally used [1]. The classical approach fixes the values of the safety factors and chooses the values of the design variables to satisfy the safety conditions. All the variables involved are then assumed to be deterministic and fixed to particular quantiles, i.e. mean value or characteristic values [2]. The probability-based approach deals directly with realistic random variables to find the global probability of failure as the basic design criterion. Obviously, the later problem is more difficult since the evaluation of the probability of failure is not an easy task, especially when the structure has more than one failure mode (multimode failure or multiple design points). In this case, analysis of the structural system is required to evaluate the safety of the structure as a whole [1]. To handle problems of this kind, the real structure is sometimes modelled by an equivalent system in such a way that all relevant failure modes can be treated [2].

The reliability analysis of plates and shells with respect to plastic collapse or to inadaptation was formulated on the basis of limit and shakedown theorems [3]. The technique was based upon an upper bound approach using the re-parameterized exact Ilyushin yield surface and nonlinear optimization procedures. Based on a direct definition of the limit state function, the non-linear problems may be efficiently solved by using the First and Second Order Reliability Methods (FORM/SORM) because shakedown analysis renders the problem time invariant [4]. In order to get the design point, a non-linear optimization was implemented. FORM/SORM match particularly well with direct plasticity methods because they render the problem time invariant.

The non-linear optimization algorithm developed in [3] is guaranteed to converge to a minimum-distance point on the limit state surface, provided that the limit state function is continuous and differentiable. However, as with any non-convex optimisation problem, it is not guaranteed that the solution point will be the global minimum-distance point when the system has more than one failure mode. This paper aims at extending the method developed in [3] for the probabilistic shakedown analysis of multimode-failure shell structures. A method to successively find the multiple design points of a component reliability problem, when they exist on the limit state surface, is presented. Each design point corresponds with an individual failure mode or mechanism. FORM and SORM approximations are applied at each design point followed by a series system reliability analysis to lead to improved estimates of the system failure probability.

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