

Extended Velocity-Pressure Enrichments for Solving Moving Interface two-Phase Flows

Azzeddine Soulaïmani, ¹, Adil Fahsi ² and Mamadou Touré ³

¹ Mechanical Engineering department, École de technologie supérieure, Montreal, Canada
, E-mail: Azzeddine.soulaïmani@etsmtl.ca

² Mechanical Engineering department, École de technologie supérieure, Montréal, Canada,
E-mail: adil.fahsi.1@ens.etsmtl.ca

³ Mechanical Engineering department, École de technologie supérieure, Montréal, Canada,
E-mail: mamadou-kabirou.toure.1@ens.etsmtl.ca

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Multi-phase flows and moving interface problems exist in nature and in technological systems. Numerical modeling of such flows faces several intrinsic difficulties, mainly due to the changes in the topology of the interface as it evolves with time, the jumps in fluid density and viscosity across the interface, and the multi-scales of time and space in the physical phenomena that may occur. Various numerical methods have been used to overcome these difficulties; for instance, interface-tracking and interface-capturing methods have been developed to describe the interface evolution. In standard finite element methods with level-set techniques, the approximation of the unknown interface is not always aligned with the grid. Standard polynomial finite element spaces have very poor approximation quality when used for discretization. For instance, in laminar flows with a high viscosity jump across the interface, the velocity profile has a kink that cannot be numerically captured if the interface is located inside the elements. With gravitational two-phase flows, it is the pressure that has a kink. Numerical discretization errors in the flow variables induce spurious currents that may be reduced by using very fine meshes. The eXtended Finite Element Method (XFEM) introduced initially by Belytschko and Black addresses these difficulties by potentially making the mesh independent of the interface geometry. In this talk, we address the issue of

the choice of the enrichment functions for the velocity and for the pressure and investigate a solution algorithm. We consider flows dominated by gravity and by the jumps in the fluid properties (viscosity and density). We advocate the use of the Taylor-Hood element so that the impact of the enrichments can then be systematically investigated without any consideration of either the stabilization technique or the iterative solver. The velocity is either un-enriched or enriched by the modified-abs function ($P2-P1$ Ridge function). The pressure is enriched by the modified-abs function, the discontinuous sign function or the discontinuous abs function. Several numerical examples will be presented showing the stability and the accuracy of the enrichments.

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