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B-Spline and reproducing polynomial particle shape functions for linear and nonlinear elasticity problems

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ABSTRACT

Both the B-Spline functions and the reproducing polynomial particle (RPP) shape functions are piecewise polynomials and have the polynomial reproducing property. The B-Spline functions are non-negative and can be constructed to be enough smooth. The reproducing polynomial particle (RPP) shape functions satisfy the Kronecker delta property at particles. Moreover, the RPP shape functions and can be constructed to be smooth up to any desired order by either raising polynomial degree or widening its support.

In this paper, the two kinds of base functions are used for solving 2-D linear and nonlinear elasticity problems. Tensor product approach is used to construct 2-D base functions. A structured grid with proper scale and implicit equations representing the boundaries of the analysis domain are used to divide the analysis domain into boundary sub-domains and internal sub-domains. Complete approximation functions used for the analysis domain can be determined based on information of sub-domains. The boundary sub-domain requires further subdivisions for high accuracy of integration if the scale is large. Essential boundary conditions can be imposed directly in regular domain and by Lagrange multipliers method, couple method or penalty method in general domain. The two kinds of base functions constructed with different parameters are used in simulation of several 2-D linear and nonlinear elasticity problems. The numerical results are compared with analytical and finite

element analysis solutions. For linear elasticity problems in general domain, the quadratic and cubic B-spline functions performs better than other B-spline functions using the present analysis method. The reproducing polynomial particle (RPP) shape functions with enough smoothness (at least C^1 continuity) and reproducing order ($\geqslant 1$) also demonstrate good approximation ability. For nonlinear elasticity analysis, the total Lagrangian (TL) approach is used to establish the solution formulations. The two kinds of base functions with proper parameters (smoothness, reproducing order and support) can catch the deformation state in simulation of large deformation of beam and hyperelastic rubber. They perform much better than shape functions of FEM in the large deformation analysis. The numerical examples show that the two kinds of base functions in combination with the present analysis method are potential to solve complicated problems in mechanics.

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