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XFEM AND STABILIZATION FOR 3D INCOMPRESSIBLE TWO-PHASE FLOWS

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Two-phase systems play an important role in chemical engineering, for example mass transport between droplets and a surrounding liquid (liquid-liquid system) or heat transfer in falling films (liquid-gas system). The velocity and pressure field are smooth in the interior of each phase, but undergo certain singularities at the interface Γ between the phases. Surface tension induces a pressure *jump* across Γ, and a large viscosity ratio leads to a *kink* of the velocity field at Γ, especially for liquid-gas systems.

If interface capturing methods (like VOF or level set techniques) are applied, the finite element grid is usually *not aligned with the interface*. Then for standard FEM the approximation of functions with such singularities leads to poor $\mathcal{O}(\sqrt{h})$ convergence. The application of suitable extended finite element methods (XFEM) provides optimal approximation properties [2, 5], essentially reducing spurious currents at the interface. Figure 1 shows the pressure jump of a static bubble using a standard and an extended finite element space.

Besides of its optimal approximation properties, unfortunately, the linear systems arising from XFEM discretizations can be highly ill-conditioned because of a deterioration of the inf-sup constant and a lack of robustness w.r.t. the relative location of the interface

Figure 1: Pressure jump of a static bubble using piecewise linear FEM (left) and suitable XFEM (right).

inside cut elements. Recently, a couple of stabilization techniques have been proposed to attenuate these drawbacks [1, 4].

In this talk we consider 3D incompressible flow simulations of such two-phase systems on adaptive multilevel tetrahedral grids. We present an XFEM space which is very suitable for the approximation of discontinuous pressure in incompressible two-phase flow problems and study the stabilization properties of a ghost penalty method applied in that context. At the end of the talk, we present application examples of 3D droplet and falling film simulations obtained by our 3D two-phase flow solver DROPS [3, 6].

REFERENCES

[1] E. Burman and P. Hansbo. Fictitious domain methods using cut elements: III. A stabilized Nitsche method for Stokes' problem. *Mathematical Modelling and Numerical Analysis*, 2013. DOI 10.1051/m2an/2013123.

[2] S. Groß and A. Reusken. An extended pressure finite element space for two-phase incompressible flows with surface tension. *J. Comput. Phys.*, 224:40–58, 2007.

[3] S. Groß and A. Reusken. *Numerical Methods for Two-phase Incompressible Flows*, volume 40 of *Springer Series in Computational Mathematics*. Springer, 2011.

[4] P. Hansbo, M.G. Larson, and S. Zahedi. A Nitsche method for a Stokes interface problem. *arXiv preprint*, 1205.5684, 2012.

[5] A. Reusken. Analysis of an extended pressure finite element space for two-phase incompressible flows. *Comput. Visual. Sci.*, 11:293–305, 2008.

[6] DROPS package for simulation of two-phase flows.
`http://www.igpm.rwth-aachen.de/DROPS/`.