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REGULARIZATION OF NONHOLONOMIC CONSTRAINTS IN MULTIBODY SYSTEMS

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Rolling contacts are usual in various technical systems. Gear wheels in gearboxes, the motion of rolling elements in roller bearings or the Euler disk [1] can be mentioned here as examples. Even the sliding of a clutch disk on an elastic support can become unstable transitioning into a rolling motion [2]. In most cases rolling motion conditions yield non- holonomic constraint equations. Usually the non-holonomic constraints can be incorporated by the method of Lagrange multipliers. This formulation leads to index-2 differential algebraic problems. Different regularization approaches [3, 4] have been developed for higher index DAEs because of the numerical drift problems inherent for usual ODE methods. In the present paper we investigate a new regularization method that is motivated by physical considerations. Pure rolling is equal to "sticking" with a kinematically repositioned contact point. Usually sticking is modeled by introducing elasticity in the contact [5]. Although constraints are mainly enforced by the elasticity in that case and the dissipative terms are necessary in order to avoid numerical oscillations in the contact. A suitable choice of the order of magnitude of the dissipative forces enhances the numerical performance of the method. The purpose of this kind of regularization is to get a consistent description for sliding, sticking and rolling contacts. Stamm [6] applied this kind of regularization to a tangential contact law, extending the classical laws of friction to distributed contacts, where the problem of indeterminacy in the sticking state is circumvented by regularization. It this work we prove the convergency of the spring-damper regularization for the so called principal damping, which is motivated by the critical damping in the linear case, and the strong damping which is of the same magnitude order as the contact stiffness.

Problem formulation and convergence theorems

Consider the MBD problem with non-holonomic constraints which can be formulated as follows:

$$\begin{aligned} \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} &= \mathbf{F}(\mathbf{q},\dot{\mathbf{q}},\mathbf{t}) - \mathbf{G}^{\mathbf{T}}(\mathbf{q})\boldsymbol{\lambda} \\ \mathbf{G}(\mathbf{q})\dot{\mathbf{q}} &= \mathbf{0} \end{aligned} \tag{1}$$

Here $\mathbf{M}(\mathbf{q})$ denotes the mass matrix of dimension $(n\times n)$, $\mathbf{G}(\mathbf{q})$ the constraint matrix of dimension $(m\times n)$ which is assumed to have the full rank m and $\mathbf{F}(\mathbf{q},\dot{\mathbf{q}},\mathbf{t})$ the column matrix of all external forces acting on the system. Note that if the functions are sufficiently smooth, the Lagrange' multiplier can be calculated explicitly:

$$\boldsymbol{\lambda} = \left(\mathbf{G}\mathbf{M}^{-1}\mathbf{G}^{\mathbf{T}}\right)^{-1}\left(\mathbf{G}_{\mathbf{q}}\dot{\mathbf{q}}\dot{\mathbf{q}} + \mathbf{G}\mathbf{M}^{-1}\mathbf{F}\right);\ \mathbf{G}_{\mathbf{q}} = d\mathbf{G}/d\mathbf{q} \tag{2}$$

Consider the corresponding regularized problem:

$$\begin{gathered}\mathbf{M}(\overline{\mathbf{q}})\ddot{\overline{\mathbf{q}}}=\mathbf{F}(\overline{\mathbf{q}},\dot{\overline{\mathbf{q}}},\mathbf{t})-\mathbf{G}^{\mathbf{T}}(\overline{\mathbf{q}})\overline{\boldsymbol{\lambda}}\\ \mathbf{G}(\overline{\mathbf{q}})\dot{\overline{\mathbf{q}}}=\dot{\mathbf{z}};\ \overline{\boldsymbol{\lambda}}=\frac{c}{\varepsilon}\mathbf{z}+\frac{d}{\varepsilon^{\kappa}}\dot{\mathbf{z}}\end{gathered} \tag{3}$$

Here ε is the small parameter describing the stiffness of the “sticking” contact, κ is the damping exponent. If $\kappa=1$ one can speak about the strong damping (it means for small ε the damping forces dominate over elastic forces in the relationship for the regularized Lagrange’ multiplier in (3)). The case $\kappa=1/2$ can be denoted as the principal damping because elastic and damping forces in the regularized contact are of the same magnitude order.

It can be shown that the solutions of the constrained (1) and regularized (3) equations converge for sufficiently small ε under certain assumptions concerning the smoothness of the introduced functions and the appropriate choice of the consistent initial conditions both for the strong and for the principal damping, i.e.

$$\begin{gathered}\left\|(\mathbf{q},\dot{\mathbf{q}})-(\overline{\mathbf{q}},\dot{\overline{\mathbf{q}}})\right\|<M_{1/2}\sqrt{\varepsilon}\ \text{for the time interval } O(1) \text{ in case of the principal damping}\\ \left\|(\mathbf{q},\dot{\mathbf{q}})-(\overline{\mathbf{q}},\dot{\overline{\mathbf{q}}})\right\|<M_{1}\varepsilon\ \text{for the time interval } O(1) \text{ in case of the strong damping}\end{gathered} \tag{4}$$

These estimates are valid at least after the short time interval in which the regularized system reaches the exponential vicinity of the constraint manifold. The estimation for the principal damping requires a certain relationship between the regularization parameters c and d in comparison with the maximal eigenvalues of the Matrix $\mathbf{GM}^{-1}\mathbf{G}^{\mathbf{T}}$. Numerical examples are presented.

Conclusions

Convergency of the visco-elastic regularization is proven for nonholonomic constraints in general form. The proof is performed for the principal damping exponent. Numerical experiments confirm the optimal performance for this choice of the regularization parameters. The described approach enables consistent modeling of sticking, sliding and rolling contacts in multibody dynamics.

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