

## **Boundary effect on the elastic field and effective elasticity of a semi-infinite solid containing particles**

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The boundary effect of one inhomogeneity embedded in a semi-infinite solid at different depths has firstly been investigated using the fundamental solutions for Mindlin's problem. Expanding the eigenstrain in a polynomial form and using the Eshelby's equivalent inclusion method, one can calculate the eigenstrain and thus obtain the elastic field. When the inhomogeneity is far from the boundary, the solution recovers Eshelby's solution. The method has been extended to a many-particle system in a semi-infinite solid, which is firstly demonstrated by the cases of two spheres. The comparison of the asymptotic form solution with the finite element models shows the accuracy and capability of this method. The solution has been used to illustrate the size and boundary effects of a representative volume element (RVE) on its effective material behavior of a simple-lattice particulate composite. The local field of a semi-infinite composite has been calculated at different volume fractions. An RVE has been taken with different sizes and depths to the surface. The effective stiffness of the RVE has been calculated under uniform loading conditions of normal or shear stress on the surface, respectively. The effective elastic moduli not only depend on the material proportion and microstructure, but also on the size and location of RVE. With the increase of the RVE size, the boundary effect decreases. The solution can be extended to other microstructures of ellipsoidal particles.