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LARGE-SCALE FREE MATERIAL OPTIMIZATION ON 3D DESIGN DOMAINS BY AN INTERIOR POINT METHOD

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Free Material Optimization (FMO) was introduced in structural optimization in [1] and [2]. Since then, extensive research and development of models, theory, and methods has been documented in e.g. [3], [4, 5], and [6]. In FMO the design parametrization allows the material tensor to vary almost freely at each point of the design domain. The only requirements imposed on the material tensor is that it must be symmetric positive semidefinite with bounded trace. Solution to FMO problems give both the optimal material distribution and the optimal local material properties, see Figure 1.

A discrete FMO formulation is obtained after partitioning the design domain into m finite elements. We consider the classical multiple load minimum compliance FMO problem stated in the matrix design variables E_i and the displacement (state) variables u_ℓ.

$$\begin{aligned} \underset{u_1,\dots,u_L,E\in\mathbb{E}}{\text{minimize}} \quad & \sum_{\ell=1}^{L} w_\ell f_\ell^T u_\ell \\ \text{subject to} \quad & A(E)u_\ell = f_\ell, \quad \forall\, \ell, \quad \sum_{i=1}^{m} Tr(E_i) \le V, \end{aligned} \tag{1}$$

where $\mathbb{E}$ denotes the set of admissible design variables

$$\mathbb{E} := \left\{ E_1,\dots,E_m \in \mathbb{R}^{6\times 6} \mid E_i = E_i^T, E_i \succeq 0, \underline{\rho} \le Tr(E_i) \le \overline{\rho},\ i = 1,\dots,m \right\}.$$

The stiffness matrix $A(E)$ is assumed to be linear in E_i and the static loads f_ℓ are assumed to be design-independent. Problem (1) can be shown to be well-posed after imposing natural assumptions on the volume bound $V > 0$ and the local bounds $0 \le \underline{\rho} < \overline{\rho} < +\infty$.

The optimization problem (1) is a non convex Semi Definite Program (SDP). This non-standard problem has many small matrix inequalities and special optimization methods have to be developed and implemented. Our objective is to show that it is possible to obtain high quality solutions to large-scale FMO problems for 3D structures using only a

(a) Design domain with boundary conditions and external load.

(b) The trace of the optimal FMO design variables.

Figure 1: An examples of a single load 3D FMO problem.

modest number of iterations. This is achieved by modifying existing primal-dual interior point methods for nonlinear programming and linear SDPs to FMO problems. The main bottleneck is the computation of the search directions. This requires the solution of a very large-scale saddle point system [7] at each iteration of the interior point method. This system becomes increasingly more ill-conditioned as the optimum is approached. We propose special purpose preconditioners combined with iterative Krylov subspace methods for the saddle-point system and numerically show that they are capable of dealing with the increasing ill-conditioning. Extensive numerical experiments confirm that the combination of techniques generates an efficient and robust method which is capable of routinely solving large-scale FMO problems on 3D design domains.

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