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GRAPH GRAMMAR FOR CONSTRUCTION OF ELIMINATION TREES FOR FAST SOLUTION OF H ADAPTED MESHES

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In this paper we present a graph grammar for construction of the elimination trees for multi-frontal direct solver algorithm working with h adaptive grids with point singularities. The graph grammar constructs such the elimination trees that the computational cost and memory usage of the direct solver algorithm are linear O(N) for both two and three dimensional problem with point singularities. The solver uses the concept of reutilization [1] constructing the elimination tree where the mesh nodes neighbouring the singularities are located at the top of the elimination tree. Such the tree enables for reutilization of the LU factorizations from the unrefined parts of the mesh each time we execute the new refinements. The graph grammar approach has been already used for modelling adaptive two and three dimensional grids [2,3]. In this paper we introduce graph grammar enabling for automatic construction of the optimal elimination trees at the same time as we perform mesh refinements, compare Figure 1. The resulting scalability of the multi-frontal solver has been tested on a sequence of two and three dimensional problems, compare Figures 2. In all the cases we outperform the state-of-the-art MUMPS solver [3]. We also discuss the possible extension of our strategy into two and three dimensional grids with other singularities, compare Figure 3. In all the cases depicted in Figure 3, except the anisotropic face refinement, we recover the linear computational cost.

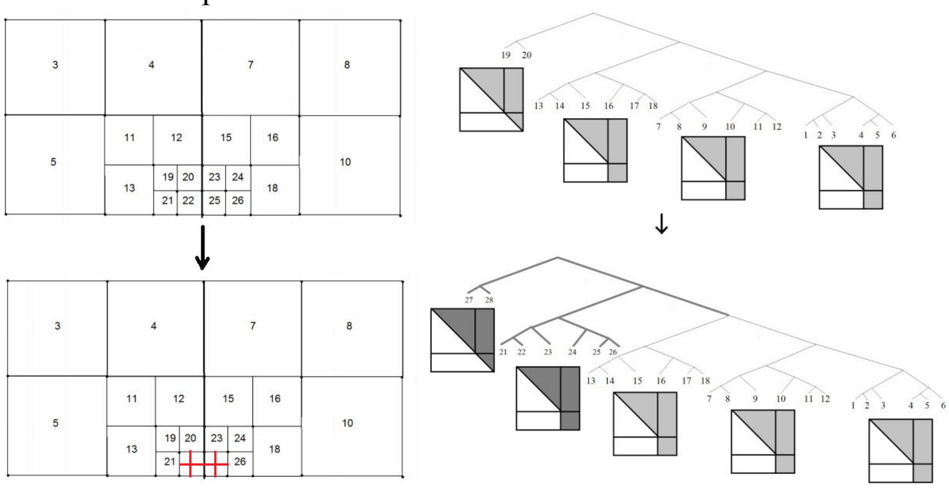

Figure 1. Transforming the elimination tree when the mesh is locally refined

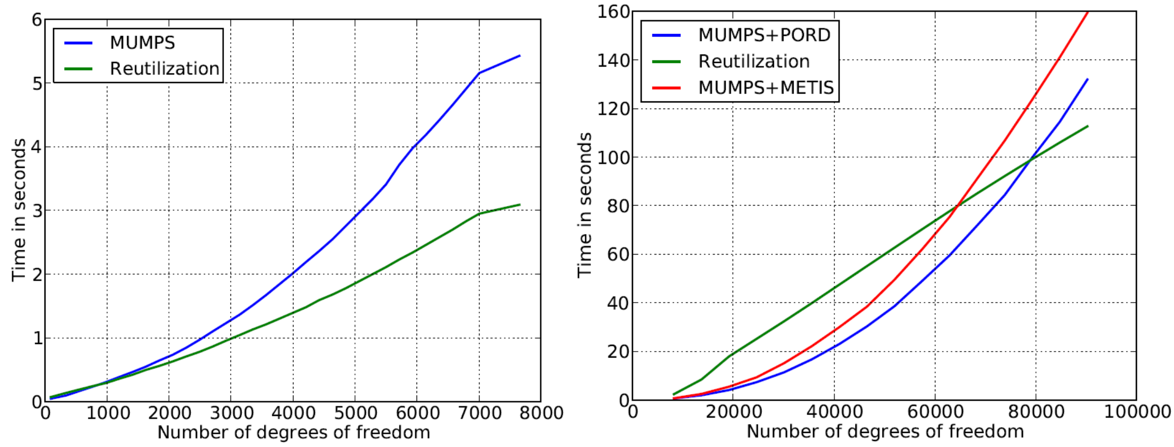

Figure 2. The scalability of our solver compared to the scalability of MUMPS solver on a sequence of 2D (left panel) or 3D (right panel) grids refined towards point singularity.

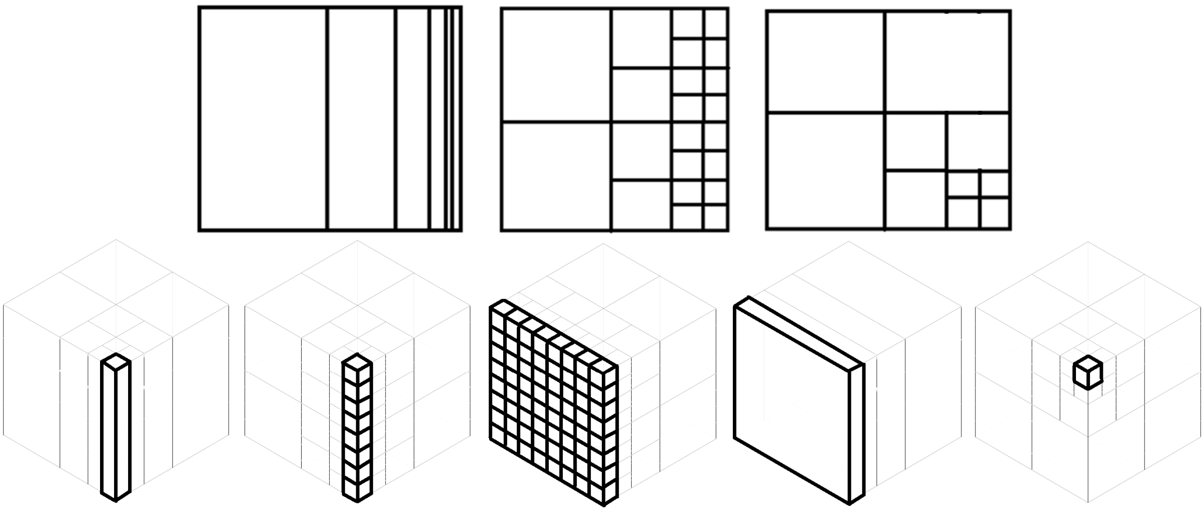

Figure 3. The three basis ways of refinement of a two dimensional rectangular element: isotropic and anisotropic edge refinements, isotropic edge plus point refinements and point refinements. The six basic ways of refinement of a three dimensional hexahedral element, isotropic and anisotropic edge and face refinements, and point refinements.

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[3] MUlti-frontal Massively Parallel sparse direct Solver MUMPS, http://graal.ens-lyon.fr/MUMPS/