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COMPARED COMPUTATIONAL PERFORMANCES OF GALERKIN APPROXIMATIONS FOR PERTURBED VARIABLE-COEFFICIENT DIFFERENTIAL EQUATIONS, ONE-DIMENSIONAL ANALYSIS

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In recent decades, the Moving Least Squares (MLS) approximation [1] has become an efficient and well-established technique for solving partial differential equations numerically. First employed by Nayroles *et al*. as the base of the Diffuse Element Method (DEM), it was later incorporated by Belytschko *et al*. to the Galerkin weak form of differential equations yielding the meshless Element-Free Galerkin Method [2]. Nowadays, the MLS-based meshless methods are an alternative to Finite Element analysis in those problems in which the presence of a mesh is a limiting factor for the advance of the computational process, for instance intensive remeshing in the simulation of fracture phenomenon.

Contributions by Hrenikoff, Courant, Argyris, Turner, Clough and Zienkiewicz [3] laid the foundation of the most widely used numerical code for engineering applications in industry today, the Finite Element Method (FEM), which is based on a topological map to discretize the equations in the domain of analysis. The $h-$refinement offers a powerful and flexible tool to deal with complex geometries, while the $p-$refinement is generally limited to Lagrange polynomials of first or second order in most of the commercial software packages.

In recent works, Bernstein polynomials [4] have been tested with success as trial functions for boundary value problems. The introduction of Bernstein expansion into a Galerkin scheme yields a spectral meshfree-type approximation with smooth globally-supported shape functions that possess interesting properties related to consistency and reproduction capability [5]-[6]. Bernstein polynomials constitute a Partition of Unity (PU) by themselves and form a complete basis of the polynomial space allowing the reproduction of any bounded continuous function by uniform convergence when the order of the basis is progressively increased.

In this context, the computational performance of meshfree MLS technique when solving the Galerkin weak form of one-dimensional perturbed variable-coefficient differential equations is tested against mesh-based FEM and spectral-type Lagrange and Bernstein PUs. The general boundary value problem considered for analysis is governed by a non-homogeneous variable-coefficient second-order differential equation with small perturbing parameter ε. This is a classic mathematical model of convection-diffusion phenomena, studied in depth with numerical approximations [7]. The four numerical techniques are subjected to experimentation in benchmark problems of the perturbation theory [8]. The analytical solutions to be approximated may exhibit boundary layer structure, which means sharp gradients of the field variable in narrow regions. The parameters swept in the analysis are the value of the small perturbing parameter ε, the number of nodes n in the discretization and the intrinsic selectable parameter d_{max} of MLS. The nodal distributions selected are patterns of equispaced nodes, therefore the thickness of the boundary layers is not a required datum for the positioning of the nodes. Accuracy, convergence and computational cost are measured and compared for each analysis case.

The main conclusion derived from the experiments is that MLS technique is able to fill the gap between FEM —the faster method, but with a limited accuracy for the reproduction of the boundary layers— and the spectral-type methods —which present excellent accuracy but are much more computationally demanding—. MLS allows to retrive very accurate results without the computational efforts associated to the generation of full matrices, bridging therefore both extremes of error vs machine time diagrams. Parameter d_{max} can be adjusted to tune the desired accuracy. Flexibility is expected to be improved by considering other selectable parameters of MLS, like the order or enrichment of the intrinsic basis and the adequate choice of the weight function.

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