11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

A VERTEX-BASED HIGH-ORDER FINITE-VOLUME SCHEME FOR THREE-DIMENSIONAL COMPRESSIBLE FLOWS ON TETRAHEDRAL MESH

Marc R.J. Charest[1], Thomas R. Canfield[2], Nathaniel R. Morgan[3], Larry D. Risinger, Jr.[4], Jacob Waltz[5] and John G. Wohlbier[6]

[1] Los Alamos National Laboratory, Los Alamos, NM, U.S.A., charest@lanl.gov
[2] Los Alamos National Laboratory, Los Alamos, NM, U.S.A., canfield@lanl.gov
[3] Los Alamos National Laboratory, Los Alamos, NM, U.S.A., nmorgan@lanl.gov
[4] Los Alamos National Laboratory, Los Alamos, NM, U.S.A., deanr@lanl.gov
[5] Los Alamos National Laboratory, Los Alamos, NM, U.S.A., jwaltz@lanl.gov
[6] Los Alamos National Laboratory, Los Alamos, NM, U.S.A., wohlbier@lanl.gov

Key words: *Numerical Algorithms, Computational Fluid Dynamics, High-Order Methods, Compressible Flows, Shock Hydrodynamics.*

High-order discretization methods for conservation laws have the potential to significantly reduce the cost of modelling physically-complex flows, but this potential is challenging to fully realize, especially for multi-dimensional unstructured mesh. It is difficult to obtain accurate discretizations of hyperbolic conservation laws, such as those governing compressible flows, that do not generate any undesirable oscillations near discontinuities or shocks [1]. One promising high-order discretization is the Central Essentially Non Oscillatory (CENO) finite-volume approach [2–4], which was demonstrated to remain both accurate and robust for a variety of physically-complex flows. This robustness is provided by a hybrid reconstruction procedure that switches between two algorithms: an unlimited high-order k-exact reconstruction in smooth regions, and a monotonicity-preserving limited piecewise linear reconstruction in regions with discontinuities. Fixed central stencils are used for both reconstruction algorithms, which makes its extension to arbitrary unstructured meshes straightforward.

The existing CENO formulations for structured [2, 3] and unstructured [4] mesh were developed for cell-centered finite-volume schemes only. In the present research, the CENO approach was extended to a vertex-based finite-volume discretization on tetrahedral mesh. The resulting algorithm was used to solve the equations governing compressible flows and analyzed in terms of accuracy and computational cost. In particular, the accuracy of the proposed scheme was examined for several function reconstructions as well as steady and unsteady idealized flow problems. Up to fifth-order accuracy was demonstrated for smooth problems, and non-oscillatory solutions were demonstrated for problems containing discontinuities. Some sample results are provided below.

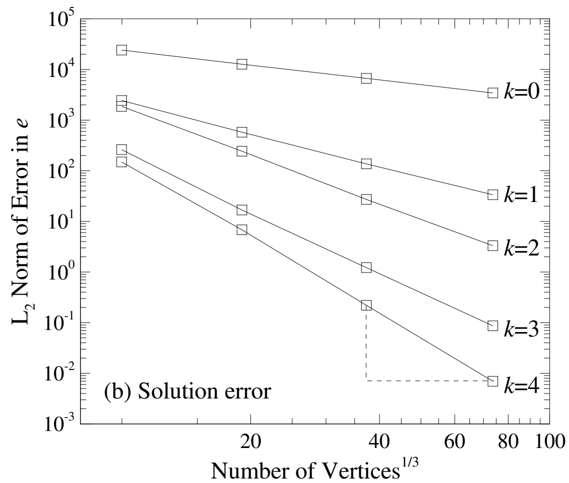

Figure 1: Effect of mesh size on solution error for smooth supersonic flow.

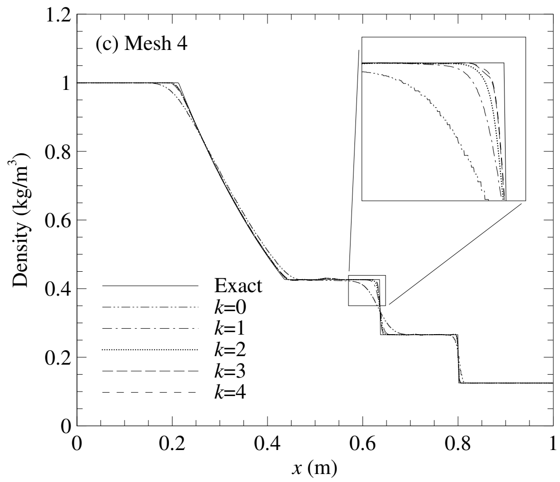

Figure 2: Predicted density profiles for the one-dimensional shock-tube problem.

The spatial accuracy of the proposed finite-volume formulation was verified for smooth flows using the method of manufactured solutions (MMS) [5]. MMS was used to derive analytical source terms that produced smooth, sinusoidal-shaped solutions of three-dimensional, supersonic, inviscid flow. The L_2 norm of the error in predicted internal energy for this problem is illustrated in Fig. 1. For all values of k considered (where k is the degree of the polynomial interpolant), the formal order of accuracy was achieved by the L_2 norm. The other norms, i.e., the L_1 and L_∞ norms, also displayed similar convergence characteristics.

A one-dimensional shock-tube problem was also investigated to demonstrate the robustness of the algorithm. The problem was solved on a rectangular domain with each half of the shock-tube at a different initial state. The predicted density at a time of $t = 2\,\mathrm{s}$ is illustrated in Fig. 2 for different values of k. The CENO method robustly handled all discontinuous features of the problem without producing any undesired oscillations in the solution.

REFERENCES

[1] A. Harten, B. Engquist, S. Osher, and S.R. Chakravarthy. J. Comput. Phys., 71(2): 231–303, 1987.

[2] L. Ivan and C.P.T. Groth. AIAA paper 2007-4323, 2007.

[3] L. Ivan and C.P.T. Groth. J. Comput. Phys., 257(1):830–862, 2013.

[4] M.R.J. Charest and C.P.T. Groth. AIAA paper 2013-2567, 2013.

[5] P.J. Roache. J. Fluids Eng., 124(1):4–10, 2001.

LANL report no. LA-UR-13-28958