11th. World Congress on Computational Mechanics (WCCM XI)
5th. European Conference on Computational Mechanics (ECCM V)
6th. European Conference on Computational Fluid Dynamics (ECFD VI)
July 20 - 25, 2014, Barcelona, Spain

A NOVEL SOLVER ACCELERATION TECHNIQUE BASED ON DYNAMIC MODE DECOMPOSITION

Niklas Andersson[1] and Lars-Erik Eriksson[2]

[1] Chalmers University of Technology, SE-41296 Gothenburg Sweden,
niklas.andersson@chalmers.se
[2] Chalmers University of Technology, SE-41296 Gothenburg Sweden,
lars-erik.eriksson@chalmers.se

Key words: *Computational Fluid Dynamics, Compressible Flows, Computing Methods, Dynamic Mode Decomposition.*

The speed up of finite-volume solvers for compressible flows is a difficult task. There are several ways to achieve solver speed-up, more or less difficult to implement and more or less suitable for implementation in a parallel, unstructured type of solver. Examples of such techniques are the multi-grid method and implicit residual smoothening. In this article, a solver acceleration technique based on Dynamic Mode Decomposition (DMD) is proposed. The technique does not depend on data or mesh structure and is thus as straightforward to implement in an unstructured parallel code as in a structured sequential code. The main idea behind the proposed method is that one can use the information in flow field modes extracted using the DMD technique to find a correction that will bring the solution closer to a steady state condition, *i.e.* the method is only applicable to steady-state problems. In the presented work the DMD-based acceleration technique has been implemented in a massively parallel block-structured finite-volume Navier-Stokes solver for compressible flows. The method has been tested on two simple one-dimensional test cases and on a turbine cascade with promising results. To the knowledge of the authors, the proposed method is not previously published in the open literature.

The Dynamic Mode Decomposition (DMD) was first introduced by Schmid [1] as a method for extracting coherent dynamic flow structures from a previously generated set of data samples. The fact that the method does not require any information about the origin of the data means that it can be applied to data obtained from for example transient numerical simulations or experiments. The DMD method is a Krylov subspace method related to the iterative Arnoldi algorithm for extraction approximate eigenvalues and eigenvectors of large matrices. The technique proposed by Arnoldi [2], also described in detail by *e.g.* Ruhe [3] and Eriksson *et al.* [4], is based on the projection of a high-dimension system matrix onto subspace of significantly lower dimension. The nature of this subspace is such that its eigenvalues represents the least damped modes of the extensively larger system matrix. One of the benefits of the DMD method over a method based directly on the Arnoldi algorithm is that it can be applied to an existing set of samples as a post processing procedure whereas an eigenmode extraction method based on the Arnoldi algorithm would require that the data set is generated as part of the eigenmode extraction procedure and thus it is impossible to use this kind of method on an existing data set.

The Generalized Minimal Residual (GMRES) technique proposed by Saad *et al.* [5] is a Krylov subspace method for solving linear systems. GMRES is related to the above-mentioned Arnoldi algorithm and has been used to speed up the convergence of flow solvers in steady state application, see *e.g.* Nigro *et al.* [6]. Since the GMRES method is a Arnoldi-based technique that may be applied for solver speed up purposes, it seems natural that it is possible to utilize the mode information that can be extracted using the DMD method to speed up the convergence of flow solvers as well. The presented DMD-based acceleration technique uses a set of stored solver flow fields to calculate a low-dimension projection of the system matrix describing the flow field development. This approximate system matrix is than used to generate a flow field correction that brings the solution closer to convergence. The flow corrections are done in a step-wise manner alongside an otherwise normal solver execution. The most efficient size and sample frequency of the generated data sets are very problem dependent and has to be found by trial and error. However, with suitable settings, the proposed DMD-based acceleration technique is able to produce significant speed up.

REFERENCES

[1] Schmid, P. J., "Dynamic mode decomposition of numerical and experimental data," *Journal of Fluid Mechanics*, Vol. 656, 2010, pp. 5–28.

[2] Arnoldi, W. E., "The principle of minimized iteration in the solution of the matrix eigenproblem," *Quart. Appl. Math.*, Vol. 9, 1951, pp. 17–29.

[3] Ruhe, A., "Rational Krylov Sequence Methods for Eigenvalue Computation," *Linear Algebra and its Applications*, Vol. 58, 1984, pp. 391–405.

[4] Eriksson, L.-E. and Rizzi, A., "Computer-Aided Analysis of the Convergence to Steady State of Discrete Approximations to the Euler Equations," *Journal of Computational Physics*, Vol. 57, 1985, pp. 90–128.

[5] Saad, Y. and Schultz, M. H., "GMRES: A Generalized Minimal Residual Algorithm for Solving Nonsymmetric Linear Systems," *SIAM J. Sci. Stat. Comput.*, Vol. 7, No. 3, 1986, pp. 856–869.

[6] Nigro, N., Sorti, M., Idelsohn, S., and Tezduyar, T., "Physics based GMRES preconditioner for Compressible and Incompressible Navier-Stokes Equations," *Comput. Methods Appl. Mech. Engrg.*, Vol. 154, 1998, pp. 203–228.