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NON-OSCILLATORY FEM FOR FLOWS OVER FLOODING AREAS AND PARTIALLY ERODIBLE BEDS

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Flows over flooding areas in estuaries and rivers are often simulated by the shallow water equations. Computational domain for an estuary/river region is partially confined by an artificial boundary connecting the area with coastal regions or with the open sea, requiring special open boundary conditions (e.g. [1]). A relevant portion of the rest of the boundary is defined by evolutionary coastlines, particularly for severe flood conditions. These coastlines call for a proper numerical algorithm for the dynamics of dry–wet interfaces. Otherwise, when sediment transport is relevant coupled transport equations are added. Coupled sediment evolution necessitates a sign–preserving method to calculate physical properties that are positive definite (the thickness of the erodible layer), given that natural beds can be constituted by erodible and/or non–erodible materials. Besides, sign–preserving property is beneficial to shrink the oscillatory behavior when computed values are near zero [2]. In our case, null total height of water determines the position of the dry–wet interface.

This work presents a sign–preserving and continuous finite element method (FEM) for coupled transport equations, incorporating the shallow water equations [4] and a sediment transport equation. The sediment transport equation interacts with the fluid flow equations by time–dependent sources for momentum. Sediment transport equation relates position of the bed–fluid interface to the divergence of sediment fluxes [3] and is formulated as a standard transient advection–diffusion equation. The bedload transport is written as convective fluxes with the effective velocity representing transport rates averaged over the local effective thickness of the erodible stratum. Otherwise, sediment avalanches, acting as a natural slope limiter, are represented as diffusive fluxes with an anisotropic, inhomogeneous diffusion coefficient depending critically on the local slope [3].

The continuous FEM is developed by integrating a high order finite element procedure with a conservative flux–correction that imposes sign–preservation, permitting simulation of flows with dry fronts without spurious mass exchanges and oscillations, as well as simulation of the evolution of layers of erodible sediment over partially non–erodible stratums. Otherwise, flux–correction is a useful tool to stabilize wiggles due to arbitrary and/or discontinuous initial conditions and to very shallow and narrow regions. This benefit is mainly attributed to the correction procedure, which can have a local behavior close to a monotone first order scheme.

Experiments concentrate on the simulation of real river dynamics and dam–breaking propagation on channels, as well as evolution of sediment layers over partially non–erodible beds. To simulate bedload transport, two models have been adopted. First, a typical formula for the transport q_s as $q_s \propto u^m$ (see e.g. [7], [6]), where u is the local velocity and m an exponent. Second, formulas of the type of Meyer-Peter-Muller (see e.g. [5]), where wall stress is computed in terms of a depth–integrated friction law.

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