11th. World Congress on Computational Mechanics (WCCM XI)
5th. European Conference on Computational Mechanics (ECCM V)
6th. European Conference on Computational Fluid Dynamics (ECFD VI)
July 20 - 25, 2014, Barcelona, Spain

ANM SUPPLEMENTED WITH POWER SERIES ANALYSIS TO EFFICIENTLY COMPUTE STEADY-STATE BIFURCATIONS IN 3D INCOMPRESSIBLE FLUID FLOWS

M. Medale[1] and B. Cochelin[2]

[1] Aix-Marseille Université, CNRS, IUSTI UMR 7343, 13013 Marseille, France, marc.medale@univ-amu.fr
[2] LMA, Centrale Marseille, UPR 7051 CNRS, Aix-Marseille Université, 13451 Marseille Cedex 20, France, bruno.cochelin@centrale-marseille.fr

Key words: *Path-following or continuation methods, Asymptotic Numerical Method, power series analysis, steady-state bifurcations, hydrodynamic instabilities.*

To understand or control some complex physical phenomena or to optimize operating conditions of industrial equipments or processes, it is of first concern to know for which range of its control parameters the system is either stable or unstable. Indeed, at critical values of these control parameters, i.e. turning points, steady or Hopf bifurcation points, the stability of the system changes [8, 6, 9], so that transitions may occur and result in qualitative and quantitative changes in the system behavior. In the computational fluid mechanics community, the classical way to determine a bifurcation diagram usually consists in performing a sequence of the following two tasks for a series of control parameter values [5]: i) compute the base state associated to a given value of the control parameters; ii) compute the linear stability of the given base state, as the system becomes unstable when the growth rate is greater than zero, whereas it remains stable in the opposite case.

The present work aims at providing a computationally efficient solution to the first of these two steps in the framework of large size algebraic systems resulting from the discretization of incompressible Navier-Stokes equations. It relies on a continuation or path-following algorithm designed to compute branches of solutions for a given range of control parameters, along with the determination of critical values and their corresponding singular solution. Among the continuation algorithms, first order predictor-corrector algorithms (Euler predictor, Newton-Raphson based corrector) with pseudo-arc-length parameterization have been widely used for decades [8, 6, 9]. Nevertheless, it turns out that step-length adaptivity may be in trouble in the vicinity of bifurcation points leading to a weak computational efficiency and sometimes lack of convergence. An alternate way to first order predictor algorithms stands in high-order predictors that have been introduced in continuation algorithms based on the Asymptotic Numerical Method (ANM) [3]. This method combines high-order Taylor series expansion, discretization technique and parameterization strategy, which results in a general and efficient non-linear solution method. It has been successfully applied in mainly solid and structural mechanics, but also in few hydrodynamics problems [2, 1, 7]. In a recent work [4] we have shown that power series analysis enables to accurately detect and compute simple bifurcation points in the course of the ANM continuation. As soon as it is detected, a new power series that recovers an optimal step length is then build in the vicinity of bifurcation points and substituted to the original one, as the latter is critically penalized in the classical ANM.

This paper focusses on the valuable use of power series analysis within the ANM continuation to compute 3D steady-state incompressible fluid flow inside a sudden expansion channel (expansion ratio $E = 3$, cross-section aspect ratio $10 \leq B \leq 24$). We have computed for the first time up to four steady symmetry breaking (pitchfork) bifurcations together with their associated bifurcated branches. The main characteristic of this 3D symmetric expansion configuration is that for a given cross-section aspect ratio the first bifurcation mode induces a top-bottom asymmetry, as in the 2D case, whereas the subsequent ones modulate the former in the span-wise direction with increasing wave numbers.

REFERENCES

[1] C. Allery, J.-M. Cadou, A. Hamdouni and D. Razafindralandy. Application of the Asymptotic Numerical Method to the Coanda effect study. *Revue Eur. Eléments Finis*, Vol. **13**, 57–77, 2004.

[2] J.-M. Cadou, M. Potier-Ferry, B. Cochelin and N. Damil. Asymptotic Numerical Method for stationary Navier-Stokes equations and with Petrov-Galerkin formulation. *Int. J. Num. Meth. Eng.*, Vol. **50**, 825–845, 2001.

[3] B. Cochelin, N. Damil and M. Potier-Ferry. *Méthode Asymptotique Numérique (in french)*. Hermès - Lavoisier, 2007.

[4] B. Cochelin and M. Medale. Power series analysis as a major breakthrough to improve the efficiency of Asymptotic Numerical Method in the vicinity of bifurcations. *J. Comput. Phys.* Vol. **236**, 594–607, 2013.

[5] H. Dijkstra *et al.* Numerical bifurcation methods and their application to fluid dynamics: Analysis beyond simulation. *Commun. Comput. Phys.*, Vol. **15**, 1–45, 2014.

[6] E. Doedel, H. B. Keller and J. P. Kernevez. Numerical analysis and control of bifurcation problems (i) bifurcation in finite dimensions. *Int. J. Bifurcation and Chaos*, Vol. **3**, 493–520, 1991.

[7] Y. Guevel, H. Boutyour and J.M. Cadou. Automatic detection and branch switching methods for steady bifurcation in fluid mechanics. *J. Comput. Phys.* Vol. **230**, 3614–3629, 2011.

[8] H. B. Keller. *Numerical solution of bifurcation and nonlinear eigenvalue problems.* Academic Press, New York, 1977.

[9] R. Seydel. *Practical bifurcation and stability analysis.* Springer-Verlag, 1994.