11th. World Congress on Computational Mechanics (WCCM XI)
5th. European Conference on Computational Mechanics (ECCM V)
6th. European Conference on Computational Fluid Dynamics (ECFD VI)
July 20 - 25, 2014, Barcelona, Spain

THE RELATIONSHIP BETWEEN THE FAST WAVE AND THE FABRIC TENSOR

Young June Yoon[1]

[1] School of Mechanical Engineering, Hanyang University,
222 Wasimni-ro, Seongdong-gu, Seoul, 133-791 Korea
yyoon@hanyang.ac.kr

Key Words: *Fabric tensor, Ultrasound, Mathematical Modeling, Elastic modulus*

The fabric tensor is a good measure to determine the anisotropy of a cancellous bone. There are three types of methods to measure the fabric tensor of a cancellous bone – the mean intercept length (MIL) [1,2], the volume orientation (VO) method [3], and the star volume distribution (SVD) method [4,5].

Ultrasound is one method to determine the mechanical properties of a cancellous bone [6,7]. It was found that the elastic modulus measured by ultrasound is almost equal to the mechanically tested and measured elastic modulus [7,8]. Recently Biot theory [9.10,11] was employed to characterize the mechanical properties of a cancellous bone. Experimentally and theoretically two waves were observed in cancellous bone ultrasound measurements [12,13]. It is believed that the fast wave follows the trabeculae, the bone matrix strut in cancellous bone, and the slow wave follows the water (or marrow) in the porous space in cancellous bone [6].

We believe that the speed of a fast wave in Biot theory is identical to the speed of sound predicted by bar equation because the elastic modulus measured by ultrasound using the bar equation is identical to the mechanically tested and measured elastic modulus [7,8]. Thus we expand the bar equation by using the fabric tensor and propose a simple equation for further experiments.

After employing the equations of fabric tensor with the elasticity tensor, we conclude that

$$E_i = E_s\left(1-\phi\right)^n\left[1+d_1\phi\left(1-II-F_i-F_i^2\right)\right].$$

Here E_i is the elastic modulus in i direction, E_s is the elastic modulus of solid matrix, d_1 is the undetermined coefficient, and ϕ is the porosity. For isotropic material, the elastic modulus E_i is simply determined by the equation $E_i = E_s\left(1-\phi\right)^n$ without considering the anisotropic behavior. Wear [13] tested 53 human calcaneus samples and found the best fit for the curve between the speed of sound (SOS) and the porosity, of which exponent n is 1.75. By employing the bar equation, we can simply conclude the relationship between the speed of sound (SOS) of the fast wave and the fabric tensor F_i,

$$v_i = \sqrt{\frac{E_s\left(1-\phi\right)^{1.75}\left[1+d_1\phi\left(1-II-F_i-F_i^2\right)\right]}{\left(1-\phi\right)\rho_s+\phi\rho_w}},$$

where ρ_s is the bone matrix density and ρ_w is the density of water. The above equation is the major result obtained in this paper and it requires further experiments to determine the fabric tensor and unknown coefficients d_1. After obtaining the fabric tensors by using a micro-CT, we can determine the unknown coefficient d_1. Then we can simply formulate the speed of sound (SOS) in terms of fabric tensors.

This work was supported by the National Research Foundation of Korea (2010-0023070).

REFERENCES

[1] T.Harrigan and R.W. Mann, Characterization of microstructural anisotropy in orthotropic materials using a second rank tensor. *J. Mat. Sci*, Vol. **19**, pp. 761-769, 1984
[2] W. J. Whitehouse, The quantitative morphology of anisotropic trabecular bone. *J. Microsc* Vol. **101**, pp. 153-168, 1974
[3] A. Odgaard, J. Kabel, B. van Rietbergen, M. Dalstra, and R. Huiskes, Fabric and elastic principle directions of cancellous bone are closely related. *J. Biomech* Vol. **30**, pp. 487-495, 1997
[4] L.M. Cruz-Orive, L.M. Karlsson, S.E. Larsen, and F. Wainschtein, Characterizing anisotropy: a new concept. *Micron Microscopica Acta* Vol. **23**, pp. 75-76, 1992
[5] L.M. Karlsson, and L.M. Cruz-Orive, Application of the start volume distribution to characterize structural anisotropy of a duplex stainless steel. In *Stereology in Materials Science: Demonstration of Some Methods*. Royal Institute of Technology, Stockholm, 1993
[6] J. Y. Rho, An ultrasonic method for measuring the elastic properties of human tibial cortical and cancellous bone. *Ultrasonics* Vol. **34**, pp. 777-783, 1996
[7] C.H. Turner, S.C. Cowin, J.Y. Rho, R.B. Ashman, and J.C. Rice, The fabric dependence of the orthotropic elastic constants of cancellous bone. *J. Biomech* Vol. **23**, pp. 549-561, 1990
[8] R.B. Ashman, J.D. Corin, and C.H. Turner, Elastic properties of cancellous bone: Measurement by an ultrasonic technique. *J. Biomech*. Vol. **20**, pp. 979-983, 1987
[9] T.J. Haire, and C.M. Langton, Biot theory: a review of its application to ultrasound propagation through cancellous bone. *Bone* Vol. **24**, pp 291-295, 1999
[10] J. L. Williams, Ultrasonic wave propagation in cancellous bone and cortical bone: Prediction of some experimental results by Biot's theory. *J. Acoustic Soc. Am.* Vol. **91**, pp 1106-1112, 1999
[11] Y. J. Yoon, J. P. Chung, C. S. Bae, and S. Y. Han, The speed of sound through trabecular bone predicted by Biot theory. *J. Biomech*. Vol. **45**, pp.716-718, 2012
[12] A. Hosokawa, and T. Otani, Ultrasonic wave propagation in bovine cancellous bone. *J. Acoustic Soc. Am.* Vol. **101**, pp 558-562, 1997
[13] K.A.Wear, A. Laib, A.P. Studber, and J.C. Reynolds, Comparison of measurements of phase velocity in human calcaneus to Biot theory. *J. Acoustic Soc. Am.* Vol. **117**, pp. 3319 – 3324, 2005